NEURAL NETWORKS AND MULTI-FRACTAL MODELLING OF NON-LINEAR COMPLEX SYSTEMS

N.S.Loboda

Odesa Hydrometeorological Institute, P.O.Box 108, Odesa-9, 65009
e-mail: glushkov@paco.net

We consider an effective approach for treating the non-linear complex systems, based on the "neural networks" and multi-fractal modelling. We apply this approach for studying the chaotic dynamics of the following systems: system of two (several) auto-generators (f.e., two semiconducting quantum generators which are connected by means of optical wave-guide) and hydrological system (fluctuations of the natural rivers annual run-off).

It is well known that many physical, biological, hydrological systems (and the dynamics of their key characteristics fluctuations) can be described as mechanical dissipative multi-body systems, which are fundamentally non-linear [1–11]. General non-linear parameter dependent dynamical dissipative systems very often have parameter ranges, in which the dynamics is chaotic [1]. Chaotic behaviour in the sense of a fully deterministic evolution of the systems in time with sensitive dependence on initial conditions, might therefore be expected to occur in the above cited systems. Dissipative non-linear systems typically have a long term behaviour which is described by an attractor in phase space. At the same time the chaotic dynamics in details is often unknown. We cannot reconstruct the original attractor that has given rise to the observed time-series. Instead, we seek an effective space where we can reconstruct an attractor from the scalar data that preserves the invariant characteristics of the original attractor. It is well known that an attractor is called a strange attractor if its dimension is non-integer, i.e. fractal. Non-linear systems or fractal objects like interfaces or time-series are characterised by their scaling property related to invariance under magnification. For uniform fractals, the scaling is uniquely described by one fractal exponent, the so-called fractal dimension. In last years a study of fractal properties of the dynamical systems is of a great interest. In this paper we consider an effective method for treating the non-linear complex systems, based on the "neural networks" and multi-fractal modelling [12, 13]). The approach enables one to get a possibility of prediction the evaluation dynamics, including the extreme phenomena in non-linear complex systems. We apply these models to treating the chaotic dynamics of the following systems: system of two (several) auto-generators (e.g., two semiconducting quantum generators which are connected by means of optical wave-guide) and hydrological system (fluctuations of the annual run-off for natural river). Within neural networks-like model we introduce a non-linear component representing the immediate and moderately retarding response and linear component representing the retarding response of the complex system. The output function Z within systems model is as follows [12, 13]:

$$Z_t = \sum_{j=1}^{J} \sum_{i=1}^{n(j)} \sum_{k=i}^{n(j)} U_{i,k}^{(j)} P_{t-i+1}^{(j)} P_{t-k+1}^{(j)} + \sum_{j=1}^{J} \sum_{i=1}^{k(j)} U_{i+n}^{(j)} P_{t-(i+n)+1}^{(j)} \quad (1)$$

where $j=1,2,\ldots, J$ is the number of independent inputs, J is the number of subsystems, $(n+1)$ is the total memory length of model; P is the matrix of the j-th input series, corresponding to the j-th sub-system; $U_{i,k}$ are the ordinates of the non-linear part of the response function, U_i are the ordinates of its linear part. The solution of the model master equation for a calibration series of N outputs values $Z_1, ..., Z_N$ can be written in vector-matrix form as [13]:

$$Z = P^{(1)}U^{(1)} + ... + P^{(N)}U^{(N)}. \qquad (2)$$

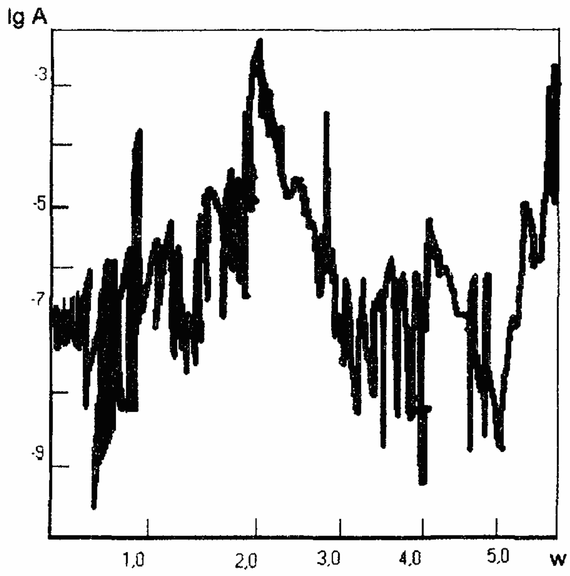

Fig.1. Spectrum of oscillations of the two generators

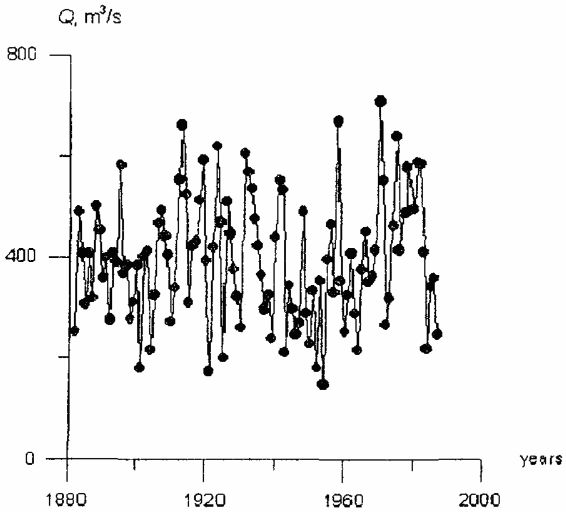

Fig.2. Time-series of the annual run-off fluctuations (the Pripyat river).

Contrary to standard models, our model allows to take into account the essential non-linearity of processes, inverse links, minimally realized elements of governing [12, 13]. Generally speaking, the systems studied (two semiconducting quantum generators which are connected by means of optical wave-guide and hydrological system) are essentially non-linear. In Fig.1 it is shown the spectrum of oscillations of the two generators (system evaluates from single-frequency sinphases regime to chaotic one; c.f. ref. [5]). In Fig 2 we present time-series of the annual run-off fluctuations (the Pripyat river, Ukraine), obtained on the basis of processing the experimental data and calculation within our neural networks stochastic model (1),(2) (cf. Ref.[13]). It should be noted that studying the annual run-off fluctuations within the multi-fractal formalism is carried at first. Non-uniform and multi-fractal objects can be more completely characterized by spectrum of $D(q)$ fractal exponent, where q is a real number (the so-called generalized dimension, where the fractal dimension is equal to $D(0)$) and the function $D(q)$ is generally referred to as multifractal spectrum [1–4, 8]. Mathematically, the general aim of the multifractal formalism is to determinate the $f(\alpha)$ singularity spectrum of measure μ. It associates the Haussdorff dimension of each point with the singularity exponent α, which gives an idea of the strength of singularity (cf. Refs.1–4, 8):

$$N_\alpha(\varepsilon) = \varepsilon^{-f(\alpha)},$$

where $N_\alpha(\varepsilon)$ is the number of boxes needed to cover the measure and ε is the size of each box. A partition function Z can be defined from this spectrum:

$$Z(q,\varepsilon) = \sum_{i=1}^{N(\varepsilon)} \mu_i^q(\varepsilon) \approx \varepsilon^{\tau(q)} \quad \text{for} \quad \varepsilon \to 0,$$

where $\tau(q)$ is a spectrum which can be obtained by Legendre transforming the $f(\alpha)$ singularity spectrum. The spectrum of generalized fractal dimensions is obtained from the spectrum $\tau(q)$:

$$D_q = \frac{\tau(q)}{(q-1)}.$$

We carried out numeric modelling of chaotic dynamics for studied systems. The analysis for the hydrological system (annual run-off fluctuations) shows that the average fractals dimensionality is 1,3–1,4. Studying the system of connected autogenerators is now in progress.

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НЕЙРОМЕРЕЖЕВЕ ТА МУЛЬТИФРАКТАЛЬНЕ МОДЕЛЮВАННЯ НЕЛІНІЙНИХ КОМПЛЕКСНИХ СИСТЕМ

Н.С.Лобода

Одеський гідрометеорологічний інститут, а/с 108, Одеса-9, 65009

Розглянуто ефективний підхід до дослідження нелінійних комплексних систем на підставі нейромережевого та мультифрактального моделювання. Досліджено нелінійні особливості в динаміці (хаос; фрактальні властивості) системи двох (декількох) автогенераторів (квантові генератори, які зв'язані оптичним хвилеводом) та гідрологічної системи (флуктуації річного річкового стоку).