THE ADIABATIC THREE-PARTICLE SHELL MODEL OF NUCLEUS

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A theoretical description of the energy spectrum of the nuclear excited slates has been carried out within the framework of the adiabatic three-particle shell model of nucleus in terms of collective variables, namely: hyperradius R, hyperangle α and conventional spherical angles (θ_i, φ_i), i=1,2. A new model is based on the assumption on the separability of the motion of the valence nucleons over fast motion on the angular variables and adiabatic (slow) motion along the hyperradius R. A convenient notion of the potential term $U_\mu(R)$ of nucleons is introduced. The efficiency of the adiabatic approach is illustrated on the example of numerical calculations of the energy spectrum of the lower excited levels of the ^{16}C, ^{18}O, ^{42}Ca, ^{58}Ni, ^{18}Ne nuclei, whose unfilled shell contains two nucleons.

1. The inclusion of the one sort of nucleons paring in the nuclear theory which play an important role in the formation of excited states of nuclei, and the angular and radial nucleon correlations study lead to the necessity of a method for calculating the stationary nuclear states that extends beyond traditional one-particle Hartree-Fock approximations [1].

To solve certain problems in the nuclear theory a hyperspheric adiabatic approach (HAA) which exceeds one-nucleon approximation has been suggested in papers [2,3].

Further development and application of this approach to the study within the framework of an adiabatic multiparticle shell nucleus model of the energy spectrum of nuclei with the inclusion of both strong and Coulomb interaction in the case of valence protons is important. It should be noted that a new so-called adiabatic model of the nucleus [2,3] is based on an assumption of the separability of valence nucleon motion over fast motion on the angular variables, i.e. at the $S^5(\Omega)$ sphere and an adiabatic (slow) motion along the hyperradius R.

In the case of $^A_Z X$ nucleus with two nucleons the nucleus description in HAA is performed in the collective variables terms whose role is played by hyperradius R and hyperangle α

$$R = (r_1^2 + r_2^2)^{1/2}, \ \alpha = arctg(r_2 / r_1) \quad (1)$$

and conventional spheric angles $\hat{r}_i = \{\varphi_i, \theta_i\}$, i=1, 2.

In the adiabatic multiparticle shell model of nucleus [2,3], as well as in the traditional multiparticle model of nucleus [4], only the residual interaction, i.e. the correlation between nucleons, is taken into account, but more thorough method is applied in the first one where the adiabatic potential term of the nucleus nucleons $U_\mu(R)$ is used.

It is known that nucleons paired correlations leading to the existence of the superfluid states of nuclei [5] are taken into account correctly and consistently in the superfluid model of nucleus [6,7] based on the secondary quantization formalism.

In this paper we suggest paired correlations between nucleons to be taken into account in the potential approach in the adiabatic shell model of the nucleus frame based upon the assumption of middle average self-consequent field of shell model and the re-

sidual interaction of nucleus valence nucleons. Thus, we assume that separate nucleons in the nucleus can be described by a set of quantum numbers n, l, j, m of the independent particle model.

2. The effective self-consequent field is modelled by Woods-Saxon potential [8] in the adiabatic shell model of nucleus

$$U_i(r_i)=\left(-V_0-2V_1\frac{N-Z}{A}t_z\right)\left(1+\exp\left(\frac{r-R_0}{a_0}\right)\right)^{-1}+$$
$$+V_k\left(\frac{1}{2}-t_z\right),\quad i=1,2, \qquad (2)$$

where in the case of valence protons the potential of Coulomb interaction V_k is modelled for simplicity as Coulomb potential of evenly charged sphere and Coulomb interaction of valence protons has a standard form.

For simplifying further calculations the residual strong interaction of valence protons is modelled by the potential with zero radius of action taking into account the repulsion of nucleons at small distances [8]

$$V_r=-V_{12}[1-g\rho(\frac{\vec{r}_1+\vec{r}_2}{2})]\delta(\vec{r}_1-\vec{r}_2). \quad (3)$$

The $\rho(\frac{\vec{r}_1+\vec{r}_2}{2})$ term in (3) effectively takes into account the repulsion of nucleons at small distances and has the sense of a total one-particle density of nucleons. A relative contribution of repulsion is defined by a constant g $(g>0)$. This choice of residual interaction essentially simplifies further the algorithm of the energy spectrum calculation because it allows its matrix elements to be calculated in an obvious analytical form, and besides, probably, does not distort the real situation, though in the future more realistic models of interaction may be considered.

The i-th nucleon spin-orbital interaction has a form:

$$W_i(r_i)=-\chi'\frac{1}{r_i}\frac{\partial U_i(r_i)}{\partial r_i},\quad i=1,2. \quad (4)$$

Thus, in the frame of a two-nucleon adiabatic shell model of nucleus the potential energy $V(R,\Omega)$ of the system under consideration in terms of collective variables has the form [2,9]

$$V(R,\Omega)=U_1(R\cos\alpha)+W_1(R\cos\alpha)(\vec{l}_1\cdot\vec{s}_1)+$$
$$+U_2(R\sin\alpha)+W_2(R\sin\alpha)(\vec{l}_2\cdot\vec{s}_2)+V_r+V_c \quad (5)$$

Note that the description of the stationary states of deformed nuclei whose average self-subsequent field is modelled by the anisotropic Woods-Saxon potential, is given in [10].

3. Let us shortly remind the basic statements of HAA [2,9]. For this purpose we consider any ${}^A_Z X$ nucleus with two nucleons above the filled shells. After separating the motion of the system inertia center, and considering the core mass as infinitely large, Schrödinguer equation for the stationary states in the hyperspheric coordinates (R,Ω) can be written as $(\hbar=m_N=1)$

$$\hat{H}\Psi(R,\Omega)=E\Psi(R,\Omega), \qquad (6)$$

where

$$\hat{H}=-\frac{1}{R^5}\frac{\partial}{\partial R}R^5\frac{\partial}{\partial R}+h(R,\Omega), \qquad (7)$$

and $h(R,\Omega)$ has the form

$$h(R,\Omega)=\Lambda^2+R^2V(R,\Omega). \qquad (8)$$

Here Λ^2 is an angular part of $\hat{H}$ at the $S^5(\Omega)$ sphere:

$$\Lambda^2=-\frac{1}{\sin^2\alpha\cos^2\alpha}\frac{\partial}{\partial\alpha}\sin^2\alpha\cos^2\alpha\frac{\partial}{\partial\alpha}+$$
$$+\frac{L_1^2}{\sin^2\alpha}+\frac{L_2^2}{\cos^2\alpha}, \qquad (9)$$

and $V(R,\Omega)$ is the potential energy (5) of the given system.

The full set $\{\Phi_\mu(R,\Omega)\} \in L_2(S^5)$ of the spectral problem solutions is taken as the hyperspheric adiabatic basis

$$h(R,\Omega)\Phi_\mu(R,\Omega) = U_\mu(R)\Phi_\mu(R,\Omega). \quad (10)$$

$U_\mu(R)$ are the nucleon adiabatic terms depending on R as the parameter for a fixed set of μ quantum numbers. 4-angle variables $\{\theta_i,\varphi_i\}$ separation is performed [2,9] by expanding $\Phi_\mu(R,\Omega)$ over the spinor spherical functions. The choice of the basis (10) allows the operators of spin-orbital interaction to be diagonalized constructively.

The expansion of the total wave function of the system $\Psi(R,\Omega)$ over a hyperspheric adiabatic $\{\Phi_\mu(R,\Omega)\}$ basis has the form

$$\Psi(R,\Omega) = R^{-5/2}\sum_\mu F_\mu(R)\Phi_\mu(R,\Omega). \quad (11)$$

The adiabatic potential terms $U_\mu(R)$ of nucleons and the corresponding $\Phi_\mu(R,\Omega)$ basis functions are found by numerical solution of the systems of differential equations over the variable α

$$\left[\frac{d^2}{d\alpha^2} - \frac{l_1(l_1+1)}{\cos^2\alpha} - \frac{l_2(l_2+1)}{\sin^2\alpha} + U_\mu(R)\right]\varphi^{(\mu)}_{\lambda_1\lambda_2 l_1 l_2}(R,\alpha) +$$

$$+ R^2 \sum_{\lambda_1'\lambda_2' l_1' l_2'} V^{\lambda_1\lambda_2 l_1 l_2}_{\lambda_1'\lambda_2' l_1' l_2'}(R,\alpha)\varphi^{(\mu)}_{\lambda_1'\lambda_2' l_1' l_2'}(R,\alpha) = 0, \quad (12)$$

for coefficients

$$\varphi^{(\mu)}_{\lambda_1\lambda_2 l_1 l_2}(R,\alpha) = \sin\alpha\cos\alpha\,\Phi^{(\mu)}_{\lambda_1\lambda_2 l_1 l_2}(R,\alpha). \quad (13)$$

The system of equations (12) is complemented by the corresponding boundary conditions [2,9] which ensure the $\varphi^{(\mu)}(R,\alpha)$ function limitation at zero and the Pauli principle fulfillment.

The energy spectrum of stationary states of nucleus within the framework of a new adiabatic three-particle model is found [2,9] by the numerical solution of the system of coupled differential equations for radial functions $F_\mu(R)$

$$\left\{-\frac{d^2}{dR^2} - \frac{1}{4R^2} + U_\mu(R) - 2E\right\}F_\mu(R) +$$

$$+\sum_{\mu'}\left\{H_{\mu\mu'}(R)F_{\mu'}(R) + Q_{\mu\mu'}(R)\frac{d}{dR}F_{\mu'}(R) + \frac{d}{dR}\left[Q_{\mu\mu'}(R)F_{\mu'}(R)\right]\right\} = 0, \quad (14)$$

where the matrix elements $H_{\mu\mu'}(R)$ i $Q_{\mu\mu'}(R)$ are defined by the formulae

$$H_{\mu\mu'}(R) = H_{\mu'\mu}(R) = \left\langle \frac{d}{dR}\Phi_\mu(R,\Omega) \middle| \frac{d}{dR}\Phi_{\mu'}(R,\Omega)\right\rangle, \quad (15)$$

$$Q_{\mu\mu'}(R) = -Q_{\mu'\mu}(R) = -\left\langle \Phi_\mu(R,\Omega) \middle| \frac{d}{dR}\Phi_{\mu'}(R,\Omega)\right\rangle. \quad (16)$$

Radial functions $F_\mu(R)$ satisfy the boundary condition

$$F_\mu(0) = F_\mu(\infty) = 0. \quad (17)$$

At numerical calculations, the $[0,\infty)$ interval over variable R is replaced by $[0,R_{max}]$ finite interval, and the system of equations (14) is limited by a finite number of equations. Adiabatic approximation corresponds to the conservation in (11) of a single term which reduces the system (14) to one equation.

An explicit form of the matrix elements of the potentials due to a strong interaction, shown in (12), is given in [9,11], and the matrix elements of the Coulomb interaction potential of valence protons are easily found in an explicit analytical form.

It should be noted that the choice of the average potential in the form of (2), and the residual interaction in the form of (3) enabled us to find its matrix elements in an explicit analytical form and, thus, to simplify considerably the algorithm of the numerical solution of system (12) with respect to $U_\mu(R)/R^2$, though in the future on can consider more realistic models of interaction. Note that in our previous papers [9-12] in order to simplify the consideration we have restricted ourselves to the valence neutrons as the valence nucleons.

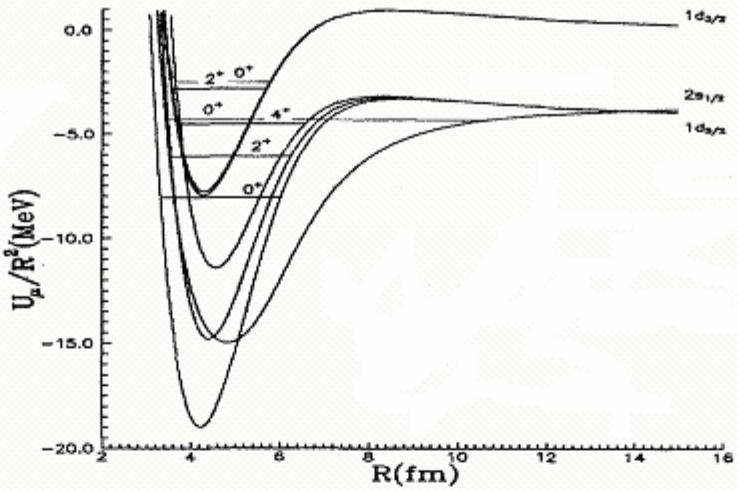

Fig. 1. The behaviour of the ^{18}O nucleus potential curves (terms) $U_\mu(R)/R^2$.

Table 1. The results of the ^{16}C, ^{18}O, ^{42}Ca, ^{58}Ni, ^{18}Ne nuclei energy excited state calculations.

Nucleus ^{A}X	Nucleon configuration	J^π	E_{exper} [13] for ^{A}X, MeV	E_{theor} for ^{A}X, MeV	E_{exper} [13] for ^{A-1}X, MeV	$U_\mu(R)/R^2$ at R=14.985 Fm, MeV
^{16}C	$1d_{5/2}\,1d_{5/2}$	0^+	0	0	−1.2177	−1.5252
	$1d_{5/2}\,1d_{5/2}$	2^+	1.7660	1.7667	−1.2177	−1.5252
	$1d_{5/2}\,1d_{5/2}$	4^+	4.1420	4.1329	−1.2177	−1.5252
	$2s_{1/2}\,2s_{1/2}$	0^+	3.0270	3.0297	−1.9577	−1.8009
	$1d_{3/2}\,1d_{3/2}$	0^+	–	5.1212	3.4393	2.0938
	$1d_{3/2}\,1d_{3/2}$	2^+	6.1090	6.1071	3.4393	2.0938
^{18}O	$1d_{5/2}\,1d_{5/2}$	O^+	0	0	−4.15	−3.8886
	$1d_{5/2}\,1d_{5/2}$	2^+	1.98207	1.980608	−4.15	−3.8886
	$1d_{5/2}\,1d_{5/2}$	4^+	3.55484	3.554252	−4.15	−3.8886
	$2s_{1/2}\,2s_{1/2}$	O^+	3.63376	3.630662	−3.27	−3.7509
	$1d_{3/2}\,1d_{3/2}$	2^+	5.25480	5.252792	0.93	0.2978
	$1d_{3/2}\,1d_{3/2}$	O^+	5.33640	5.334850	0.93	0.2978
^{42}Ca	$1f_{7/2}\,1f_{7/2}$	0^+	0	0	−8.3	−5.8
	$1f_{7/2}\,1f_{7/2}$	2^+	1524.70	1524.71	−8.3	−5.8
	$1f_{7/2}\,1f_{7/2}$	4^+	2752.41	2752.45	−8.3	−5.8
	$1f_{7/2}\,1f_{7/2}$	6^+	3189.33	3189.99	−8.3	−5.8
^{58}Ni	$2p_{3/2}\,2p_{3/2}$	0^+	0	0	−10.2650	−11.0960
	$2p_{3/2}\,2p_{3/2}$	2^+	1.4545	1.4563	−10.2650	−11.0960
	$2p_{1/2}\,2p_{1/2}$	0^+	2.9424	2.9426	−9.1524	−9.8904
	$1f_{5/2}\,1f_{5/2}$	0^+	3.5309	3.5310	−9.4965	−9.9832
	$1f_{5/2}\,1f_{5/2}$	2^+	3.8983	3.9018	−9.4965	−9.9832
	$1f_{5/2}\,1f_{5/2}$	4^+	4.2990	4.3043	−9.4965	−9.9832
^{18}Ne	$1d_{5/2}\,1d_{5/2}$	0^+	0	0	−0.5921	−0.60
	$1d_{5/2}\,1d_{5/2}$	2^+	1.8873	1.8875	−0.5912	−0.60
	$1d_{5/2}\,1d_{5/2}$	4^+	3.3762	3.3765	−0.5901	−0.60
	$1d_{3/2}\,1d_{3/2}$	0^+	3.5763	3.5766	1.9102	4.04
	$1d_{3/2}\,1d_{3/2}$	2^+	3.6164	3.6165	1.9106	4.04
	$2s_{1/2}\,2s_{1/2}$	0^+	4.5900	4.5889	−0.1446	−0.11

4. Let us illustrate the HAA method efficiency within a framework of a three-particle shell model on the example of the calculation of lower excited state energy spectrum in the ^{16}C, ^{18}O, ^{42}Ca, ^{58}Ni, ^{18}Ne nuclei, which have two valence nucleons in the unfilled shell. Therefore, in the case ^{18}Ne nucleus besides the strong nucleon interaction we shall take into account the contribution of Coulomb interaction of valence protons. According to the asymptotic behaviour of $U_\mu(R)/R^2$ terms at $R\to\infty$, considered in detail in [9,11], the energy spectrum calculations have been carried out in the following order. The Wood-Saxon potential parameters were chosen in such a way that when solving equation (12) the $U_\mu(R)/R^2$ terms asymptotically at $R\to\infty$ according to [9,11], reached the corresponding isotope level, i. e. the nucleus which contains one nucleon less than ${}^{A}_{Z}X$ nucleus. The obtained Woods-Saxon potential parameter data well coincided with parameters, which are contained in works of another authors [8].

Further, having defined the potential parameters (2), (3) for each nucleus we found the potential $U_\mu(R)/R^2$ terms and the basis functions $\Phi_\mu(R,\Omega)$ by numerical solution of equation (12) with corresponding boundary conditions taking into account only the $V^{j_1 j_2 J_0}_{l_1 l_2 l_1' l_2'}(R,\alpha)$ diagonal matrix elements. In Fig. 1 the behaviour of the potential $U_\mu(R)/R^2$ terms for each of the nucleon state configurations are shown. The found $U_\mu(R)/R^2$ terms were substituted in equation (14) over R in Bhorn-Oppenheimer approximation (i.e. disregarding non-diagonal $H_{\mu\mu'}(R)$ and $Q_{\mu\mu'}(R)$ matrix elements) with the inclusion of boundary conditions (17); the energy spectrum of low-lying excited nucleus states was found numerically. The energy of the ground state of the corresponding nucleus was taken as zero.

The theoretical calculations of the energy levels of low-lying excited nuclei states within the framework of three-particle shell model are presented in Table 1, and straight lines represent their locations at the adiabatic nucleon potential terms of a corresponding nuclei. For example in case of the ^{18}O nucleus, the positions of the energy levels are presented in Fig. 1. In this format zero is the detachment energy of two nucleons of the corresponding nucleus.

The comparison of the obtained theoretical calculations on the excited nuclei energy states with the existing experimental data [13,14] indicates their fairly good coincidence.

Thus, the introduced adiabatic three-particle shell model in the potential approach within the framework of Schrödinger equation allows one to describe adequately the effect of nucleon paring, their angular and radial correlations leading, in particular, to the formation of the superfluid states due to the strong interaction. The analysis and the detailed description of the superfluid nuclear states in the adiabatic approach is the subject of our further work.

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АДІАБАТИЧНА ТРИЧАСТИНКОВА ОБОЛОНКОВА МОДЕЛЬ ЯДРА

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Проведено теоретичний опис енергетичного спектру збуджених ядерних станів в рамках адіабатичної тричастинкової оболонкової моделі ядра в термінах колективних змінних, а саме гіперрадіуса R, гіперкута α і звичайних сферичних кутів (θ_i, φ_i), $i=1,2$. Нова модель грунтується на припущенні про відокремлюваність руху валентних нуклонів на швидкий рух по кутових змінних і адіабатичний (повільний) рух вздовж гіперрадіуса R. Введено зручне для опису поняття потенціального терма нуклонів $U_\mu(R)$. Ефективність адіабатичного підходу проілюстровано на прикладі чисельних розрахунків енергетичного спектру нижніх збуджених рівнів ядер ^{16}C, ^{18}O, ^{42}Ca, ^{58}Ni, ^{18}Ne, незаповнена оболонка яких містить два нуклони.