DETERMINATION OF THE RESISTIVITY TENSOR FOR MULTITERMINAL MEASUREMENTS IN THE CASE OF *C*-AXIS CURRENT

P.P. Popovich

Uzhgorod State University, Voloshin St. 54, Uzhgorod, 88000, Ukraine

The method, based on the solution of Laplace's equation for computing resistivity tensor by multiterminal method, was developed to system with large anisotropy. The current is injected in the direction of the c-axis.

Introduction

One of the common characteristic of solids is their resistivity and its dependence on the temperature. The shape of R-T curves give information about type of material (metal, semiconductor, superconductor), gives information about phase transitions (i.e. superconductor-to-metal) and helps to obtain parameters like activation energies etc. That is why the methods of determination of resistivity play vital role.

The simplest methods for resistivity determination (two- and four-probe) need specially prepared samples, they are useful for isotrope materials. In case of anisotrope solids we deal with resistivity tensor, that can be simplified to the diagonal form

$$\rho = \begin{bmatrix} \rho_a & 0 & 0 \\ 0 & \rho_b & 0 \\ 0 & 0 & \rho_c \end{bmatrix},$$

if contacts follow the crystallographic axis.

One of the commonly-used methods for anisotropy materials is carried out by Montgomery [1]. In this case four contacts are placed in the corners of the sample, and the current and voltage leds are periodically changed. This method gives the resistivities in the measured plane (ρ_a,ρ_b).

R.Busch et al [2] developed a method for the high temperature superconductors (HTSC) where the anisotropy between in-plane and out-plane is big.Six contacts are attached to the single crystal (Fig 1.). In this approach the resistivity tensor is uniaxial:

$$\rho = \begin{bmatrix} \rho_{ab} & 0 & 0 \\ 0 & \rho_{ab} & 0 \\ 0 & 0 & \rho_c \end{bmatrix},$$

where $\rho_{ab} = \sqrt{\rho_a \rho_b}$.

G.A.Levin et al [3] have shown that the approximation previsious method is inaccurate and improved it for the case of big anisotropy ($\Gamma = \exp\left[\gamma \frac{\pi D}{L}\right] >> 1$).

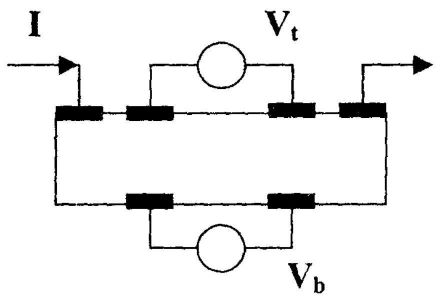
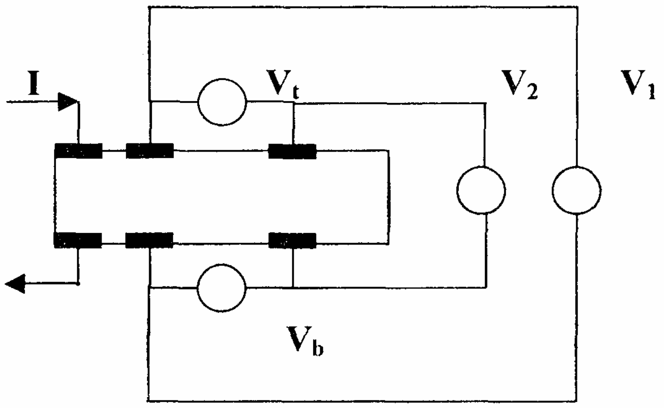


Fig.1. Contact positions for Busch or Levin metods.

The latter two models were developed for the case of a-b plane current injection. Under external magnetic field HTSC transport measurements give information about the vortex motion phenomenon.For c-axis injection Gonzalez et al [3] developed a method to obtain independently ρ_c.

Unfortunately their method work only at a given distance of voltage and current contacts and specimen length, and it given no information about the another two components of the resistivity tensor. In order to improve this two shortcomings of the latter method [3], we performed calculations for a general six terminal configuration.

Fig.2. Contact positions for our method.



The model

In the sixth terminal configuration (Fig.2.) the current I injected to the sample through contacts 1-4. On the top and bottom surfaces we measure the V_t and V_b voltages respectivelly. If we assume that $\rho_a \approx \rho_b$, we can use the two-dimensional Laplace equation in quasistatic limit, namely:

$$\frac{1}{\rho_{ab}}\frac{\partial^2 V(x,z)}{\partial x^2}+\frac{1}{\rho_c}\frac{\partial^2 V(x,z)}{\partial z^2}=0$$

The boundary conditions are:

$$\left.\frac{\partial V}{\partial x}\right|_{x=0}=0 \qquad \left.\frac{\partial V}{\partial x}\right|_{x=L}=0$$

An appropriate solution is given by expression:

$$V(x,z)=V_0 z+\sum_{n=1}^{\infty}V_n\cos\left(\frac{n\pi x}{L}\right)\sinh\left(\frac{n\pi\gamma\left(\frac{D}{2}-z\right)}{L}\right)$$

where $\gamma=\left(\frac{\rho_c}{\rho_{ab}}\right)^{\frac{1}{2}}$, L and D are length and thickness of the sample. From the last boundary condition we obtain:

$$V(x,z)=\frac{I\rho_c z}{bL}-\sum_{n=1}^{\infty}\frac{2I\sqrt{\rho_{ab}\rho_c}\cos\left(\frac{n\pi x}{L}\right)\sinh\left(\frac{n\pi\gamma\left(\frac{D}{2}-z\right)}{L}\right)}{nb\pi\cosh\left(\frac{n\pi\gamma D}{2L}\right)},$$

where I is a current, b is the width of the sample in the y direction.

In the contact arrangment like on the Fig.2. the measured potentional differences on the top and bottom, V_t and V_b be equal.

Calculating of the voltages V_1 and V_2 are simple:

$$V_1=V(x_1,D)-V(x_2,D),$$
$$V_2=V(x_1,0)-V(x_2,0)$$

Note that if $\gamma \gg 1$ the the $\tanh\left(\frac{n\pi\Gamma}{2}\right)\longrightarrow 1$, and

$$\sum_{n=1}^{\infty}\frac{\cos\left(\frac{n\pi x_0}{L}\right)}{n}=-\frac{1}{2}\ln\left[2\left(1-\cos\left(\frac{x_0\pi}{L}\right)\right)\right]$$

We can easily obtain the equatoions for V_1 and V_2:

$$V_1=\frac{I\rho_c D}{bL}-\frac{2I(\rho_{ab}\rho_c)^{\frac{1}{2}}}{b\pi}\ln\left(2\left(1-\cos\left(\frac{\pi x_1}{L}\right)\right)\right)$$

$$V_2=\frac{I\rho_c D}{bL}-\frac{2I(\rho_{ab}\rho_c)^{\frac{1}{2}}}{b\pi}\ln\left(2\left(1-\cos\left(\frac{\pi x_2}{L}\right)\right)\right)$$

From these equation we can obtaqin the elements of the tensor:

$$\rho_{ab}=\frac{\pi^2 bD(U_{c1}-U_{c2})^2}{4IL\left\{U_{c1}+\frac{(U_{c1}-U_{c2})\ln\left[2\left(1-\cos\left(\frac{\pi x_1}{L}\right)\right)\right]}{\ln\left[\frac{1-\cos\frac{\pi x_2}{L}}{1-\cos\frac{\pi x_1}{L}}\right]}\right\}\left(\ln\left[\frac{1-\cos\frac{\pi x_2}{L}}{1-\cos\frac{\pi x_1}{L}}\right]\right)^2},$$

$$\rho_c=\frac{bL}{ID}\left(U_{c1}+\frac{(U_{c1}-U_{c2})\ln\left[2\left(1-\cos\left(\frac{\pi x_1}{L}\right)\right)\right]}{\ln\left[\frac{1-\cos\frac{\pi x_2}{L}}{1-\cos\frac{\pi x_1}{L}}\right]}\right).$$

Conclusions

In this work we present the approach to compute the resistivity tensor from measured voltage and current in multiterminal method.We simplify problem to uniaxial tensor that is why we can work with 2D Laplace's equation. A new feature here is the c-axis current injection. The condition of the usability of this method is the big effective anisotropy factor Γ. The results have been compared by the experimental data measured on $Bi_2Sr_2CaCu_2O_8$ single crystals [5,6,7].

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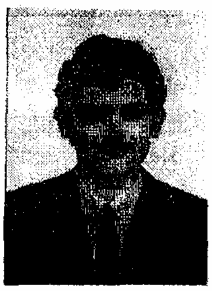
ВИЗНАЧЕННЯ ТЕНЗОРА ЕЛЕКТРОПРОВІДНОСТІ ДЛЯ МУЛЬТИТЕРМІНАЛЬНИХ ВИМІРЮВАНЬ У ВИПАДКУ ІНЖЕКЦІЇ СТРУМУ ПО ОСІ *C*

П.П. Попович

Ужгородський державний університет, 88000, м. Ужгород, вул. Волошина, 54

Для випадку великої анізотропії було розроблено метод, що базується на розвязку рівняння Лапласа, для визначення тензора електропровідності з мультітермінальних вимірювань. Струм інжектується по напрямку осі *c*.



Павло Павлович Попович – молодший науковий співробітник Інституту фізики і хімії твердого тіла.
Народився у 1974 р. Закінчив фізичний факультет УжДУ в 1996 р.