# CONFIGURATION INTERACTION METHOD FOR DESCRIBING SPIN –ORBITAL EFFECTS IN THE QUARK–ANTIQUARK SYSTEMS

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The screened quasi-relativistic potential is used for describing spin-orbit splitting in $^3P_J$ waves of quark-antiquark system. Fermi-Breit equation is solved numerically in configuration interaction approximation. This approximation takes into account the mixing of partial waves up to order eight and corrects substantially perturbation calculations. The nature of potential's Lorentz transformation property is elucidated. Very good quantitative results for $b\bar{b}$ and $c\bar{c}$ quarkonia and quite acceptable qualitative numbers are obtained for $u\bar{u}$.

Today it seems evident that quark potential model gives a rather good description of spin-averaged mass spectrum of hadrons, considered as a system of quarks [1]. The nonrelativistic Cornell as well as oscilator potential with Coulumb-like one gluon exchange and power-law confinement terms is used. In this work we try to extend this approach to incorporating the second order spin-terms in two-quark Fermi-Breit equation for evaluating the spin-orbit splitting. Instead of calculating as usual spin-terms in first-order perturbation approximation the expansion of the total wave function into a basic set of unperturbed solutions up to the fifth order of configurationally interacting states is carried out.

The problem of mass splitting also can shed some light on the Lorentz nature of the quark potential.

The main problem is to clarify some aspects of these questions in framework of configuration interaction approach (CI, [2]). This method does not need the assumption that the coupling constant is to be small, the assumption which is requred by perturbation method. Since in quark potential case it is not so the use of perturbation method looks very dubious.

Let us suggest that the static quark potential has vector and scalar property of Lorentz transform:

$$V_{NR}(r) = V_V(r) + V_S(r) \qquad (1).$$

Following many authors we asume admixture vector-scalar potential

$$\left.\begin{aligned} V_V(r) &= -\frac{\alpha_S}{r} + \varepsilon\frac{g^2}{6\pi}\cdot\frac{\left(1-e^{-\mu\cdot r}\right)}{\mu} \\ V_S(r) &= (1-\varepsilon)\cdot\frac{g^2}{6\pi}\cdot\frac{\left(1-e^{-\mu\cdot r}\right)}{\mu} \end{aligned}\right\} \qquad (2)$$

where $\varepsilon$ is mixing constant.

The choise of potential itself is dictated by the consideration of most accurate description of averaged mass spectrum and

here Chikovani-Jenkovsky- Paccanoni (CJP) potential seems to be the best [3].

In addition, as Gerasimov has pointed out [4] the spin-orbit term has to be of short range, as is indicated by quantum chromodinamics (QCD), the condition which is evidently satisfied by CJP potential;

In what follows we shall use the screened potential [3], which proved to be very good in describing the spin-averaged mass-spectrum of both bosons and baryons as quark systems [5-8] and which secures the nesessary fall-of the spin-dependent forces.

Let us start with two-body Fermi-Breit equation. We shall use nuclear system of units $\hbar = c = 1$, $1GeV = \frac{5.068}{1Fm}$. The Hamiltonian of the system has the form

$$\hat{H} = \hat{H}_0 + \hat{W}, \qquad (3)$$

where

$$\hat{H}_0 = -\frac{1}{2m}\Delta + \left(-\frac{\alpha_S}{r} + \frac{g^2}{6\pi}\frac{\left(1-e^{-\mu r}\right)}{\mu}\right) \qquad (4),$$

$m$ is the reduced mass. The eigenfunctions $\varphi_n$ and eigenvalues $E_n^0$ are calculated numerically.

$$\hat{W} = \hat{H}_{LS} + \hat{H}_{ST} \qquad (5),$$

where spin-dependent potentials are given by:spin-orbit interaction

$$\hat{H}_{LS} = \frac{\hat{\vec{L}}\cdot\hat{\vec{S}}}{4m_1^2 m_2^2}\frac{1}{r}\times$$

$$\times\left\{\left[\left(m_1+m_2\right)^2 + 2m_1 m_1\right]\frac{dV_V}{dr} - \left(m_1^2+m_2^2\right)\frac{dV_S}{dr}\right\} \qquad (6)$$

$$\vec{L} = \vec{r}\times\vec{p}, \quad \vec{S} \equiv \vec{S}_1 + \vec{S}_2$$

tensor terms

$$\hat{H}_T = \frac{1}{12m_1 m_2}\left[\frac{1}{r}\frac{dV_V}{dr} - \frac{d^2V_V}{dr^2}\right]\cdot S_{12} \qquad (7)$$

$$S_{12} = \frac{4}{(2l+3)(2l-1)}\times\left[\vec{S}^2\cdot\vec{L}^2 - \frac{3}{2}\vec{L}\cdot\vec{S} - 3\left(\vec{L}\cdot\vec{S}\right)^2\right]$$

Now, we consider Fermi-Breit equation

In addition to above indicated terms in Fermi-Breit Hamiltonian present are the $\vec{S}_1\cdot\vec{S}_2$ (spin-spin) and relativistic correction term (of $p^4$-order). Some authors (like [1]) indicate that the latter is important for calculating mass spectra, other authors (like [9]) argue that it shifts all the results by a constant. In our case we believe that for SL-mass difference they will not contribute.

$$\left(-\frac{1}{2m}\Delta + \left(-\frac{\alpha_S}{r} + \frac{g^2}{6\pi}\frac{\left(1-e^{-\mu r}\right)}{\mu}\right) + W\right)\Psi(\vec{r}) = E\,\Psi(\vec{r}) \qquad (8)$$

Here we suggest to use CI approach which was previously very successfully applied in atomic physics [2]. The essence of this approximation is that the total wave function $\Psi(\vec{r})$ is expanded in set of eigenfunnctions $\varphi_n$ of the unperturbed Hamiltonian $\hat{H}_0$, that is

$$\Psi(\vec{r}) = \sum_n a_n\varphi_n(\vec{r}). \qquad (9)$$

After substituting (9) into (8) and using eigenvalue $E_n^0$, we obtain the system of equations for $a_n$ which have to be truncated for resonably large $n$

$$\left.\begin{array}{l} a_1\left(E-E_1^0+W_{11}\right)-a_2W_{12}-a_3W_{13}-\ldots-a_nW_{1n}=0 \\ -a_1W_{21}+a_2\left(E-E_2^0+W_{22}\right)-a_3W_{13}-\ldots-a_nW_{2n}=0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ -a_1W_{n2}-a_2W_{n2}-a_3W_{n3}-\ldots+a_n\left(E-E_n^0+W_{nn}\right)=0 \end{array}\right\}, \qquad (10)$$

where

$$W_{ij}=\left\langle\varphi_i\left|W\right|\varphi_j\right\rangle \qquad .(11)$$

Both the basic functions $\varphi_i$ and matrix elements $W_{ij}$ are calculated numerically. Nontrivial solution there will be only if the determinant of this system vanishes

$$\begin{vmatrix} E_1^0-E+W_{11} & W_{12} & \ldots & W_{1n} \\ W_{21} & E_2^0-E+W_{22} & \ldots & W_{2n} \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ W_{n1} & W_{n2} & \ldots & E_n^0-E+W_{nn} \end{vmatrix}=0 \qquad (12)$$

The equations (12) can be solved by diagonalizing the matrix for E. The system of (10,12) is called CIA method. This is good way of finding eigenvalues $E_n$. This procedure goes far outside of perturbation method.

The CIA method turned out to be extremaly succesfull in atomic physics. In calculating atomic structure it allowed to increase the precision of calculating energy levels by one order. In the scattering processes it allowed to reveal fine resonance structure in scattering cross-sections due to formation of autoinizing states. So we expect that its applications will be even more important in strong interaction, where the perturbation method is evidently not corect. The tecnique of application of CIA is quite complicated, since it needs to handle the matrices of large dimensions. In current work we used the code elaborated by O. Zatsarinny[10]

In this work we have applied the above described method for calculating $P-$wave «fine»-splitting of $b\bar{b}$, $c\bar{c}$ and $u\bar{u}$ systems to ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$ levels. In our case the correspondind operators $H_{LS}, H_T$ will be of the form

$$W_{LS}=\frac{1}{2m^2}\frac{1}{r}\left[3\frac{\alpha_S}{r^2}+(4\varepsilon-1)\frac{g^2}{6\pi}e^{-\mu r}\right]\vec{L}\cdot\vec{S} \qquad (13)$$

$$W_T=\frac{1}{12m^2}\left[3\frac{\alpha_S}{r^3}+\left(\frac{1}{r}+\mu\right)\cdot\varepsilon\cdot\frac{g^2}{6\pi}e^{-\mu r}\right]\cdot S_{12} \qquad (14)$$

Important is that all parameters except $\varepsilon$ are taken from [3,8], where excellent description of bottomonium and charmonium spectra were obtained. Moreover as it was shown in [8] the same parameters give good masses of $\rho-$meson trajectories. Actually the value $\frac{g^2}{6\pi}=0.3GeV^2, \mu=0.054GeV$, $\alpha_S$ was taken in accordance to QCD. The only adjustable parameter was $\varepsilon$. As mentioned above all calculations were carried out numerically. Special code was constructed for this purpose. The calculations were extended to eights order in (9) ( see tables, rows 1-8), i.e. until the difference between the results did not go below several MeV level.

Table 1. $b\bar{b}$ -system, $\alpha_S=0.3, \varepsilon=0.25, m_b=5.05GeV$

| | $\Delta M$ | 1 *MeV* | 2 *MeV* | 3 *MeV* | 4 *MeV* | 5 *MeV* | 6 *MeV* | 7 *MeV* | 8 *MeV* | *EXP.*[11] *MeV* |
|---|---|---|---|---|---|---|---|---|---|---|
| $\chi_{b2}(0^+(2^{++}))-\chi_{b1}(0^+(1^{++}))$ | $1^3P_2-1^3P_1$ | 14.7 | 17.7 | 19.1 | 19.9 | 20.5 | 20.9 | 21.1 | 21.4 | 21.3 |
| $\chi_{b1}(0^+(1^{++}))-\chi_{b0}(0^+(0^{++}))$ | $1^3P_1-1^3P_0$ | 19.8 | 24.7 | 27.2 | 28.9 | 30 | 30.9 | 31.6 | 32.2 | 32.1 |
| $\chi_{b2}(0^+(2^{++}))-\chi_{b0}(0^+(0^{++}))$ | $1^3P_2-1^3P_0$ | 34.6 | 42.4 | 46.4 | 48.9 | 50.5 | 51.8 | 52.7 | 53.5 | 53.4 |
| $\chi_{b2}(0^+(2^{++}))-\chi_{b1}(0^+(1^{++}))$ | $2^3P_2-2^3P_1$ | — | 9.7 | 12.3 | 13.7 | 14.5 | 15 | 15.4 | 15.7 | 13.3 |
| $\chi_{b1}(0^+(1^{++}))-\chi_{b0}(0^+(0^{++}))$ | $2^3P_1-2^3P_0$ | — | 12.3 | 15.9 | 18 | 19.5 | 20.5 | 21.3 | 21.9 | 23.1 |
| $\chi_{b2}(0^+(2^{++}))-\chi_{b0}(0^+(0^{++}))$ | $2^3P_2-2^3P_0$ | — | 22.1 | 28.3 | 31.7 | 33.9 | 35.5 | 36.8 | 37.7 | 36.4 |

Table 1. $c\bar{c}$ -system, $\alpha_S=0.386, \varepsilon=0.2, m_c=1.675GeV$

| | $\Delta M$ | 1 *MeV* | 2 *MeV* | 3 *MeV* | 4 *MeV* | 5 *MeV* | 6 *MeV* | 7 *MeV* | 8 *MeV* | *EXP.*[11] *MeV* |
|---|---|---|---|---|---|---|---|---|---|---|
| $\chi_{c2}(0^+(2^{++}))-\chi_{c1}(0^+(1^{++}))$ | $1^3P_2-1^3P_1$ | 37.41 | 43.01 | 45.26 | 46.47 | 47.21 | 47.72 | 48.08 | 48.36 | 45.64 |
| $\chi_{c1}(0^+(1^{++}))-\chi_{c0}(0^+(0^{++}))$ | $1^3P_1-1^3P_0$ | 59.26 | 73.16 | 81.04 | 86.46 | 90.53 | 93.77 | 96.45 | 98.73 | 95.43 |
| $\chi_{c2}(0^+(2^{++}))-\chi_{c0}(0^+(0^{++}))$ | $1^3P_2-1^3P_0$ | 96.67 | 116.17 | 126.29 | 132.93 | 137.75 | 141.48 | 144.53 | 147.09 | 141.07 |

Table 1. $u\bar{u}$ -system, $\alpha_S=0.52, \varepsilon=0.135, m_u=0.33GeV$

| | $\Delta M$ | 1 *MeV* | 2 *MeV* | 3 *MeV* | *EXP.*[11] *MeV* |
|---|---|---|---|---|---|
| $a_2(1^-(2^{++}))-a_1(1^-(1^{++}))$ | $1^3P_2-1^3P_1$ | 6.3 | 11 | 12.8 | 58.2 |
| $a_1(1^-(1^{++}))-a_0(1^-(0^{++}))$ | $1^3P_1-1^3P_0$ | 234.8 | 321.3 | 385.9 | 277.3 |
| $a_2(1^-(2^{++}))-a_0(1^-(0^{++}))$ | $1^3P_2-1^3P_0$ | 241.1 | 332.3 | 398.7 | 335.5 |

Let us make the following conclusions:

1. The results for heavy quarkonium are quite good for values $\varepsilon=0.2-0.25$. For light quarkonium the results are worse, which means that more careful relativistic effects have to be taken into account.
2. The value of $\varepsilon$ indicates that confinement has prevailingly scalar character this conclusion do not contradict other autors [12].
3. As it follows from (13) at $\varepsilon=0.25$ $\left((4\varepsilon-1)=0\right)$ and the contribution of confinement vanishes totally. Maybe exactly this circumstans was the reason that some authors stated the pure one-gluon character of SL-splitting.
4. The first column in tables correspond to pure perturbation approach. It is clearly seen that this approach gives only rough qualitative estimate, but the results are drastically improving with switch on the CIA expansion. We believe that the use of CIA in quark physics has bright future.

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# ЗАСТОСУВАННЯ МЕТОДУ НАКЛАДАННЯ КОНФІГУРАЦІЙ ДО ОПИСУ СПІН–ОРБІТАЛЬНОЇ ВЗАЄМОДІЇ У КВАРК–АНТИКВАРКОВИХ СИСТЕМАХ

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У даній роботі в рамках потенціальної моделі дано опис тонкого розщеплення $^3P_J$-станів у двокваркових системах з використанням екранованого потенціалу.Для розв'язку цієї задачі вперше застосовано метод накладання конфігурацій, який добре зарекомендував себе у атомній фізиці. Отримані результати добре узгоджуються з експериментальними даними. Зроблено спробу дослідити Лоренц структуру конфайментної частини потенціалу міжкваркової взаємодії.