ON THE LONGITUDINAL ELECTROMAGNETIC WAVES

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The exact solution of the Maxwell equations with gradient-like sources is found. The interpretation of this solution containing not only transversal but also longitudinal electromagnetic waves is given.

The investigation of longitudinal electromagnetic waves in connection with the Maxwell equations with gradient-like sources was started in [1]. Here we add to these results the procedure of search for a solution of such kind of the Maxwell equations and our interpretation of this solution.

Let us remind that the original Maxwell equations with sources for electromagnetic field strengths in the flat Minkovski space in an arbitrarily fixed inertial reference frame have the form

$$\partial_0\vec{E} - curl\vec{H} = \vec{j},\ \partial_0\vec{H} + curl\vec{E} = 0,\ \mathrm{div}\ \vec{E} = \rho,\ \mathrm{div}\vec{H} = 0,$$
$$\vec{E} \equiv (E^i),\ \vec{H} \equiv (H^i),\ \vec{j} \equiv (j^i). \tag{1}$$

Here $\vec{E}, \vec{H}$ and $\vec{j}$ are vectors of electric and magnetic field strengths and of electric current density, respectively (E^i, H^i, j^i, i=1,2,3, being the Cartesian coordinates of these vectors), ρ is some charge density; all the quantities depend on $x \equiv (x^\mu) = (x^0, \vec{x}) \in R^4; x^0 \equiv ct; \partial_\mu \equiv \partial / \partial x^\mu$.

Below we consider the case of generalized electrodynamics in which the Maxwell equations (1) contain not only the electric but the magnetic sources too. We obtain here the rezults for both the generalized and traditional electrodynamics. In the terms of the 3-component complex function

$$\vec{\mathcal{E}} \equiv (\mathcal{E}^i) = \vec{E} - i\vec{H} \tag{2}$$

being used long ago by many authors (see, e.g., [2-11]), which, as $(\vec{E}, \vec{H})$ pair , is called the electromagnetic field, the generalized Eqs. (1) have the form

$$\partial_0\vec{\mathcal{E}} - i curl\vec{\mathcal{E}} = \vec{j}, \quad \mathrm{div}\vec{\mathcal{E}} = \rho \tag{3}$$

Here we consider the special case when in Eqs. (3) all the components j^μ of the 4-current

$$j \equiv (j^\mu) = (\rho, j):\ \ \rho = j^0, \vec{j} = (j^i) \tag{4}$$

are defined by one (scalar) function φ according to the formula

$$j_\mu = -\partial_\mu\varphi:\ \rho = -\partial_0\varphi,\ \vec{j} = -\mathrm{grad}\ \varphi. \tag{5}$$

Due to the understandable reasons the current (5) is called the gradient-like one (or the gradient-like source). In terms of 4-component quantity

$$\mathcal{E} \equiv (\mathcal{E}^\mu):\ \mathcal{E}^0 = E^0 - iH^0 \equiv \varphi,\ \vec{\mathcal{E}} \equiv (\mathcal{E}^i) = \vec{E} - i\vec{H}, \tag{6}$$

Eqs. (3) with gradient-like sources (5) can be rewritten in the manifestly covariant form [12]

$$Q_{\mu\nu} \equiv \partial_\mu \mathcal{E}_\nu - \partial_\nu \mathcal{E}_\mu + i\varepsilon_{\mu\nu\rho\sigma}\partial^\rho \mathcal{E}^\sigma = 0, \quad \partial_\mu \mathcal{E}^\mu = 0 \tag{7}$$

or, alternatively, in the form

$$\hat{M}\mathcal{E} = \begin{vmatrix} \partial_0 & i\partial_3 & -i\partial_2 & \partial_1 \\ -i\partial_3 & \partial_0 & i\partial_1 & \partial_2 \\ i\partial_2 & -i\partial_1 & \partial_0 & \partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & -\partial_0 \end{vmatrix} \begin{vmatrix} \mathcal{E}^1 \\ \mathcal{E}^2 \\ \mathcal{E}^3 \\ \mathcal{E}^0 \end{vmatrix} = 0. \tag{8}$$

The tensor $Q_{\mu\nu}$ in (7) has 3 independent components only (it follows from the antisymmetricity of $Q_{\mu\nu}$ and from the properties of the Levi-Chivitta tensor $\varepsilon_{\mu\nu\rho\sigma}$), so that the system (7) is the system of four independent equations coinciding with equations (3), the equation (8) being a matrix form of the equations (3) or (7).

The rewriting of Eqs. (3) in the forms (7) or (8) is useful first of all for investigations of symmetry and some other properties of these equations (see our papers [12-15]). Here we note only the following.

It follows from (7) or (8) that every component $\mathcal{E}^\mu$ of 4-component function (6) obeys the d'Alembert equation, $\partial^\nu \partial_\nu \mathcal{E}^\mu = 0$. This means, as a particular consequence, that the gradient-like current (5) obeys the inhomogenity equation, $\partial_\mu j^\mu = 0$. And if the function is real (i.e., if H^0=0) than the current $j\mu$ is real too. If φ is a complex function (i. e.,$H^0 \neq 0$) than

$$\operatorname{Re} j = (\rho_E, \vec{j}_E), \quad \operatorname{Im} j = (\rho_M, \vec{j}_M) \tag{9}$$

where subindices E, M mark the electric (magnetic) charge $\rho_{E,M}$ and current $\vec{j}_{E,M}$ densities.

Below the most general case is considered when, generally speaking, $\operatorname{Im}\varphi \neq 0$, although in every step one can put

$$\operatorname{Im}\varphi = 0 \Rightarrow \operatorname{Im} j = 0 \Rightarrow \rho_M = \vec{j}_M = 0. \tag{10}$$

It is worthwhile to emphasize that even in the case (10), the real scalar function φ generates electric currents $\vec{j}_E$ and charges ρ_E which themselves cause the appearance of the electromagnetic field. And the function $\varphi = \mathcal{E}^0$ in Eqs. (8) (real or complex) can be either an arbitrarily fixed one (but obeying the equation $\partial_\mu \partial^\mu \varphi = 0$) or even a field variable called below as scalar (in general, complex) field.

To ensure the consideration on the level of conventional axiomatic approach being mathematically well-defined, one can suppose that Eqs. (7)=(8) are defined in the generalized Schwartz function space $S_4^* = (S(R^4) \times C^4)$ (i. e., in the space of 4-component complex-valued quantities $f=(f_\mu)$, each component f_μ of which being a continuous functional over the Schwartz space $S(R^4)$ of test functions). This assumption enables one to solve Eqs. (8) by the Fourier method, and one can find the general solution of these equations with an arbitrary gradient-like current by straightforward calculations, without appealing to the electromagnetic potentials.

Without going into the details of well-known Fourier method we note only, that for the Fourier transform $\tilde{\mathcal{E}}$ the following equation is valid

$$M(k)\tilde{\mathcal{E}}(k) = 0, \quad k \in R^4, \quad \tilde{\mathcal{E}} \equiv (\tilde{\mathcal{E}}^\mu) \tag{11}$$

which is a homogeneous system of four equations for the four generalized functions

$\mathcal{E}^{\mu} \in S(R^4)^*$. The calculation of the corresponding determinant leads to the result:

$$\det M(k) \equiv \det \begin{vmatrix} k_0 & k_1 & k_2 & k_3 \\ k_1 & k_0 & ik_3 & -ik_2 \\ k_2 & -ik_3 & k_0 & ik_1 \\ k_3 & ik_2 & -ik_1 & k_0 \end{vmatrix} \equiv (k_0^2 - \vec{k}^2), \qquad (12)$$

Finally, one can easily obtain that Eqs. (8) have 4 linearly independent particular solutions, and the general solution is the superposition of these particular solutions. Having made the suitable choice for these solutions, we have found the expression

$$\mathcal{E}(x) = \int d^3k \left[\left(c^1 u_1 + c^3 u_2\right) e^{-ikx} + \left(c^2 u_1 + c^4 u_2\right) e^{ikx} \right], \quad kx = \omega t - \vec{k}\vec{x}, \quad \omega = \left|\vec{k}\right|, \qquad (13)$$

for the general solution of the Maxwell equations (8) with gradient-like sources (the natural system of units $\hbar = c = 1$ is used), where c^{α} are the amplitudes as functions of 3-momentum $\vec{k}$ (or even as generalized functions, $c^{\alpha} \in S(R^3)^*$),

$$u_1 = \begin{vmatrix} \vec{e}_- \\ 0 \end{vmatrix}, \; u_2 = \begin{vmatrix} \vec{e}_0 \\ 1 \end{vmatrix}, \; \vec{e}_0 \equiv \frac{\vec{k}}{\omega}, \qquad (14)$$

and 3-component vectors $\vec{e}_{\mp,0}$ are the eigenvectors of the quantummechanical helicity operator for the photon (as the particle with unit spin):

$$\hat{h}\vec{e}_h = h\vec{e}_h, \quad \hat{h} \equiv \frac{\vec{s}\vec{k}}{\omega}, \quad h = \mp 1, 0; \quad \vec{e}_+ = \vec{e}_-^{\,*}, \qquad (15)$$

$$s^1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{vmatrix}, \; s^2 = \begin{vmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{vmatrix}, \; s^3 = \begin{vmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}. \qquad (16)$$

The formula (13) represents the exact solution of the Maxwell equations with specific kind of inhomogenity determined by the gradient-like sources (the unusual magnetic sources can be easily put equal to zero; in this case we have $c^3 = c^4$). Therefore it is an important result even from the purely mathematicaly point of view.

To visualize the result we rewrite the formula (13) separatelly for $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$ and $\varphi = \mathcal{E}^0$:

$$\mathcal{E}(x) = \int d^3x \left[\left(c^1 e^{-ikx} + c^2 e^{ikx}\right) \vec{e}_- + \left(c^3 e^{-ikx} + c^4 e^{ikx}\right) \vec{e}_0 \right], \qquad (17)$$

$$\varphi(x) = \int d^3x \left(c^3 e^{-ikx} + c^4 e^{ikx}\right). \qquad (18)$$

As it is seen, the general solution of the Maxwell equations (3) with an arbitrary gradient-like current (5) contains both transverse and longitudinal parts,

$$\vec{\mathrm{E}}(x)=\vec{\mathrm{E}}^{tr}(x)+\vec{\mathrm{E}}^{lon}(x). \qquad (19)$$

Of course, in the case

$$\varphi=0 \Rightarrow \rho=\vec{j}=0,\ c^3=c^4=0, \qquad (20)$$

the solution of the Maxwell equations (3) contains only a superposition of the transverse waves $\vec{e}_-\exp(\mp ikx)$ (left-hand circular polarized photons which are a basis for the irreducible D(0,1) representation of the Lorentz group SL(2,C)). Nonvanishing gradient-like currents cause only the appearance of longitudinal waves $(\vec{k}/\omega)\exp(\mp ikx)$. Moreover, if the scalar field φ (and, consequently, the gradient-like current) related to the electromagnetic field $\vec{\mathcal{E}}=\vec{E}-i\vec{H}$ according to the Maxwell equations (7)=(8), is nonzero in an asymptotically big space-time region, than the longitudinal waves propagate in the same asymptotically big space-time region. It should be stressed that all the assertions of this paper are related to the case of space, where the electric and magnetic permeabilities ε=μ=1 (absence of medium).

In the case of nontrivial functions ε and μ, i. e. in the case of inhomogeneous medium, the field φ interacting with the electromagnetic field $\vec{\mathcal{E}}=\vec{E}-i\vec{H}$ (via the gradient-like current (5)) is appeared to be so closely related to the field $\vec{\mathcal{E}}=\vec{E}-i\vec{H}$ that from φ=0 it follows that $\vec{\mathcal{E}}=\vec{E}-i\vec{H}=0$. Such case is considered in our earlier papers [16-19] dealing with the hydrogen atom model in the framework of classical electrodynamics only.

Let us mention that the electrodynamics with magnetic currents and charges considered here is some natural generalization of conventional Maxwell's electrodynamics, i. e., in general, we considered unusual electrodynamics here. Nevertheless the existence of longitudinal electromagnetic waves does not depend on the presence of magnetic currents and charges. Let us emphasize that in the case (10), when the magnetic sources are put equal to zero and, therefore, $c^3=c^4$, the longitudinal electromagnetic waves still exist. We have in this case $\vec{H}^{lon}=0$ but $\vec{E}^{lon}\neq 0$. This fact is evident after rewriting the solutions (17), (18) in the terms of real electromagnetic field strengths $(\vec{E},\vec{H})$:

$$\vec{E}(x)=\int d^3k\sqrt{\frac{\omega}{2(2\pi)^3}}\begin{Bmatrix}[c^1\vec{e}_1+c^2\vec{e}_2+(c^3+c^4)\vec{e}_3]e^{-ikx}+\\ [c^{*1}\vec{e}_2+c^{*2}\vec{e}_1+(c^{*3}+c^{*4})\vec{e}_3]e^{ikx}\end{Bmatrix}, \qquad (21)$$

From (21) one can easily find

$$\vec{E}^{lon}(x)=\int d^3k\sqrt{\frac{2\omega}{(2\pi)^3}}(c^3e^{-ikx}+c^{*4}e^{ikx})\vec{e}_3, \qquad (22)$$

Therefore, the longitudinal electric waves can exist within the framework of ordinary Maxwell's electrodynamics (without magnetic monopoles) in the case, when the electric currents and charges have specific gradient-like form. Brief and preliminary information about these rezults was given by us in [20,21].

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ПРО ПОЗДОВЖНІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ

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Одержано точний розв'язок рівнянь Максвела зі струмами градієнтного типу. При інтерпретації одержаного розв'язку отримуємо не лише поперечні, але й поздовжні електромагнітні хвилі.