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CLASSIFICATION OF NON-SINGULAR MANIFOLDS IN THE SPACE M(1,4)×R(u) INVARIANT UNDER ONE- AND TWO- DIMENSIONAL NON-CONJUGATE SUBALGEBRAS OF THE LIE ALGEBRA OF THE POINCARÉ GROUP P(1,4)

Using the classification of one- and two- dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group P(1,4) into classes of isomorphic subalgebras, we have classified non-singular manifolds in the space M(1,4)×R(u) invariant under these subalgebras.

Key words: non-singular manifolds, Minkowski space, Poincaré group P(1,4), non-conjugate subalgebras of the Lie algebra, classification of subalgebras.

Introduction

In many cases, mathematical models of various processes of the nature can be described applying differential equations (linear or nonlinear) in the spaces of different dimensions and different types (Euclidean, non-Euclidean, and etc.). The majority of these equations have non-trivial symmetry groups (see, for example, [1, 2, 3, 4, 5, 6, 7, 8]).

Indeed, there are many differential equations, to solve different problems of theoretical and mathematical physics (see, for example, [6, 9, 10, 11, 12, 13, 14, 15, 16, 17]), which are invariant under the generalized Poincaré group P(1,4). The group P(1,4) is a group of rotations and translations of the five-dimensional Minkowski space M(1,4). Continuous subgroups of the group P(1,4) have been described in [18,19, 20] .

To investigate some of the above mentioned differential equations as well as other differential equations defined in the space M(1,4)×R(u) [21] and invariant under the group P(1,4) or under its non-conjugate subgroups, we can use non-singular manifolds in the space M(1,4)×R(u) invariant under the non-conjugate subgroups of the group P(1,4). Here, and in what follows, R(u) is a real number axis of the depended variable u. However, the possibilities of these applications as well as the results obtained will depend on properties of these manifolds. Properties of the manifolds depend on the structure of non-conjugate subgroups of the group P(1,4). Therefore, the investigation of the connection between the structure of non-conjugate subgroups of the group P(1,4) and the properties of corresponding to them non-singular invariant manifolds are important from different points of view.

The objective of the paper is the classification of non-singular manifolds in the space M(1,4)×R(u) invariant under one- and two-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4).

The classification of the low-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4) has been performed in [22].

By now, using the results of [22], we have classified the non-singular manifolds in the space M(1,4)×R(u) invariant under one- and two- dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4).

To show the results obtained, we have to consider the Lie algebra of the group $P(1,4)$.

The Lie algebra of the Poincaré group P(1,4) and its representation

The Lie algebra of the group $P(1, 4)$ is defined by the 15 basis elements $M_{\mu\nu} = -M_{\nu\mu}$, $\mu,\nu = 0,1,\dots,4$, and P_μ, $\mu = 0,1,\dots,4$,

that satisfy the commutation relations

$$[P_\mu, P_\nu] = 0,\ [M_{\mu\nu}, P_\sigma] = g_{\nu\sigma}P_\mu - g_{\mu\sigma}P_\nu,$$
$$[M_{\mu\nu}, M_{\rho\sigma}] = g_{\mu\sigma}M_{\nu\rho} + g_{\nu\rho}M_{\mu\sigma} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}$$

where $g_{\mu\nu}$, $\mu,\nu = 0,1,\dots,4$, is the metric tensor with components $g_{00} = -g_{11} = -g_{22} = -g_{33} = -g_{44} = 1$ and $g_{\mu\nu} = 0$, if $\mu \neq \nu$.

Let us consider the following representation of the Lie algebra of the group P(1,4):

$$P_0 = \frac{\partial}{\partial x_0},\quad P_1 = -\frac{\partial}{\partial x_1},\quad P_2 = -\frac{\partial}{\partial x_2},$$
$$P_3 = -\frac{\partial}{\partial x_3},\ P_4 = -\frac{\partial}{\partial x_4},\ M_{\mu\nu} = x_\mu P_\nu - x_\nu P_\mu$$
$$\mu,\nu = 0,1,\dots,4.$$

We pass from $M_{\mu\nu}$ and P_μ to the linear combinations

$$G = M_{04},\ L_1 = M_{23},\ L_2 = -M_{13},\ L_3 = M_{12},$$
$$P_a = M_{a4} - M_{0a},\ C_a = M_{a4} + M_{0a},\ (a = 1,2,3),$$
$$X_0 = \frac{1}{2}(P_0 - P_4),\ X_k = P_k,\ (k = 1,2,3),$$
$$X_4 = \frac{1}{2}(P_0 + P_4).$$

In the present paper, we use the complete list of non-conjugate (up to $P(1, 4)$ -conjugation) subalgebras of the Lie algebra of the group P(1, 4) given in [23].

Classification of non-singular manifolds in the space M(1,4)×R(u) invariant under one-dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group P(1,4)

The non-singular manifolds in the space M(1,4)×R(u) invariant under one-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4) can be written as

$$F\left(J_1, J_2, \dots, J_{t_0}\right) = 0,$$

where F is a smooth function of its arguments and $\{J_1, J_2, \dots, J_{t_0}\}$ are functional bases of invariants (FBI) of the one-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4). More details about the non-singular manifolds invariant under the local Lie groups of point transformations G_r^{n+m} can be found in [1, 3].

As we see, in order to construct these manifolds, we need FBI in the space M(1,4)×R(u) of one-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4). Earlier, we studied FBI [24, 25] in the space M(1,4) for non-conjugate subgroups (the conjugation was considered under the proper ortochronous group P(1,4)) of the group P(1,4).

However, while constructing these FBI, it turned out that different non-conjugate subalgebras of the Lie algebra of the group P(1,4) might have the same ones. Consequently, there is no one-to-one correspondence between non-conjugate subalgebras of the Lie algebra of the group P(1,4) and their respective FBI. Moreover, some of the FBI (which are of the same dimension) may be equivalent. More details on this theme can be found in [26, 27]. Our aim is to obtain only non-equivalent FBI. Recently, using the criterion of equivalency [26, 27], we constructed the non-equivalent FBI in the space M(1,4)×R(u), which were invariant under the non-conjugate subgroups (the conjugation was considered under the group P(1,4)) of the group P(1,4). The knowledge of these non-equivalent FBI allowed us to construct those non-singular manifolds in the space M(1,4)×R(u) invariant under non-conjugate subalgebras of the Lie algebra of the group P(1,4).

In this section we restrict ourselves to the consideration of the non-singular manifolds in the space M(1,4)×R(u) invariant only under one-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4). To classify these manifolds we have used the classification of low-dimensional subalgebras of the Lie algebra of the group P(1,4), which was performed in [22]. According to this classification, all one-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4) belong to the same class, which are denoted by A_1. More details about the classification of the real low-dimensional Lie algebras into classes of isomorphic algebras can be found in [28, 29, 30].

Proposition 1 [22].

There exist 20 non-conjugate subalgebras of the type A_1 of the Lie algebra of the group P(1,4).

Consequently, there exist 20 non-singular manifolds in the space M(1,4)×R(u) invariant under the non-conjugate subalgebras of type A_1 of the Lie algebra of the group P(1,4).

Below, for each non-conjugate subalgebra of the type A_1 of the Lie algebra of the group P(1,4), we write its bases elements and corresponding non-singular invariant manifold in the space M(1,4)×R(u).

1. $\langle P_3 \rangle$:
$$F\left(x_1, x_2, x_0+x_4, \left(x_0^2-x_3^2-x_4^2\right)^{1/2}, u\right)=0;$$

2. $\langle G \rangle$:
$$F\left(x_1, x_2, x_3, \left(x_0^2-x_4^2\right)^{1/2}, u\right)=0;$$

3. $\langle L_3 - P_3 \rangle$:
$$F\left(x_0+x_4, \left(x_0^2-x_3^2-x_4^2\right)^{1/2}, \left(x_1^2+x_2^2\right)^{1/2}, \arctan\frac{x_1}{x_2}+\frac{x_3}{x_0+x_4}, u\right)=0;$$

4. $\langle L_3 + \lambda G, \lambda > 0 \rangle$:
$$F\left(x_3, \left(x_0^2-x_4^2\right)^{1/2}, \left(x_1^2+x_2^2\right)^{1/2}, \ln\left(x_0+x_4\right)+\lambda\arctan\frac{x_1}{x_2}, u\right)=0;$$

5. $\langle L_3 \rangle$:
$$F\left(x_0, x_3, x_4, \left(x_1^2+x_2^2\right)^{1/2}, u\right)=0;$$

6. $\left\langle L_3 + \frac{1}{2}\left(P_3+C_3\right) \right\rangle$:
$$F\left(x_0, \left(x_1^2+x_2^2\right)^{1/2}, \left(x_3^2+x_4^2\right)^{1/2}, x_1x_4-x_2x_3, u\right)=0;$$

7. $\left\langle L_3 + \frac{\lambda}{2}\left(P_3+C_3\right), 0<\lambda<1 \right\rangle$:
$$F\left(x_0, \left(x_1^2+x_2^2\right)^{1/2}, \left(x_3^2+x_4^2\right)^{1/2}, \lambda\arctan\frac{x_1}{x_2}-\arctan\frac{x_3}{x_4}, u\right)=0;$$

8. $\langle X_0 + X_4 \rangle$:
$$F\left(x_1, x_2, x_3, x_4, u\right)=0;$$

9. $\langle X_4 - X_0 \rangle$:
$$F\left(x_0, x_1, x_2, x_3, u\right)=0;$$

10. $\langle X_4 \rangle$:
$$F\left(x_0+x_4, x_1, x_2, x_3, u\right)=0;$$

11. $\langle P_3 - 2X_0 \rangle$:
$$F\left(x_1, x_2, \left(x_0+x_4\right)^2+4x_3, x_0-x_4+\frac{1}{6}\left(x_0+x_4\right)^3+x_3\left(x_0+x_4\right), u\right)=0;$$

12. $\langle P_3 - X_1 \rangle$:
$$F\left(x_2, x_0+x_4, \left(x_0^2-x_3^2-x_4^2\right)^{1/2}, x_1-\frac{x_3}{x_0+x_4}, u\right)=0;$$

13. $\langle G + \alpha X_1, \alpha > 0 \rangle$:
$$F\left(x_1-\alpha\ln\left(x_0+x_4\right), x_2, x_3, \left(x_0^2-x_4^2\right)^{1/2}, u\right)=0;$$

14. $\langle L_3 - P_3 + 2\alpha X_0, \alpha > 0\rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, 2\alpha \arctan\frac{x_1}{x_2} - x_0 - x_4, \left(x_0 + x_4\right)^2 + 4\alpha x_3, 2\left(x_0 + x_4\right)^3 + +12\alpha^2\left(x_0 - x_4\right) + 12\alpha x_3\left(x_0 + x_4\right), u\right) = 0;$$

15. $\langle L_3 + \lambda G + \alpha X_3, \lambda > 0, \alpha > 0\rangle$:

$$F\left(\left(x_0^2 - x_4^2\right)^{1/2}, \left(x_1^2 + x_2^2\right)^{1/2}, \alpha \ln\left(x_0 + x_4\right) - \lambda x_3, \alpha \arctan\frac{x_1}{x_2} + x_3, u\right) = 0;$$

16. $\langle L_3 + \alpha\left(X_0 + X_4\right), \alpha > 0\rangle$:

$$F\left(x_3, x_4, \left(x_1^2 + x_2^2\right)^{1/2}, \alpha \arctan\frac{x_1}{x_2} - x_0, u\right) = 0;$$

17. $\langle L_3 + \alpha X_3, \alpha > 0\rangle$:

$$F\left(x_0, x_4, \left(x_1^2 + x_2^2\right)^{1/2}, \alpha \arctan\frac{x_1}{x_2} + x_3, u\right) = 0;$$

18. $\langle L_3 + 2X_4\rangle$:

$$F\left(x_0 + x_4, x_3, \left(x_1^2 + x_2^2\right)^{1/2}, 2\arctan\frac{x_1}{x_2} - x_0 + x_4, u\right) = 0;$$

19. $\left\langle L_3 + \frac{\lambda}{2}\left(P_3 + C_3\right) + \alpha\left(X_0 + X_4\right), 0 < \lambda < 1, \alpha > 0\right\rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_3^2 + x_4^2\right)^{1/2}, x_0 - \alpha \arctan\frac{x_1}{x_2}, \lambda x_0 - \alpha \arctan\frac{x_3}{x_4}, u\right) = 0;$$

20. $\left\langle L_3 + \frac{1}{2}\left(P_3 + C_3\right) + \alpha\left(X_0 + X_4\right), \alpha > 0\right\rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_3^2 + x_4^2\right)^{1/2}, x_1 x_4 - x_2 x_3, x_0 - \alpha \arctan\frac{x_3}{x_4}, u\right) = 0.$$

Classification of non-singular manifolds in the space M(1,4)×R(u) invariant under two - dimensional non-conjugate subalgebras of the Lie algebra of the Poincaré group P(1,4)

In this section we present the non-singular manifolds in the space M(1,4)×R(u) invariant under two-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4). To classify these manifolds, we used the classification of low-dimensional subalgebras of the Lie algebra of the group P(1,4), which was performed in [22]. According to this classification, all two-dimensional non-conjugate subalgebras of the Lie algebra of the group P(1,4) belong to two different (non-isomorphic) classes, which are denoted by $2A_1$ and A_2 .

First, we present the non-singular manifolds in the space M(1,4)×R(u) invariant under the two-dimensional non-conjugate subalgebras of type $2A_1$ (Abelian subalgebras) of the Lie algebra of the group P(1,4).

Proposition 2 [22].

There exist 38 non-conjugate subalgebras of the type $2A_1$ (Abelian subalgebras) of the Lie algebra of the group P(1,4).

Consequently, there exist 38 non-singular manifolds in the space M(1,4)×R(u) invariant under the two-dimensional non-conjugate subalgebras of type $2A_1$ of the Lie algebra of the group P(1,4).

Below, for each two-dimensional non-conjugate subalgebra of the type $2A_1$ of the Lie algebra of the group P(1,4), we write its bases elements and corresponding non-singular invariant manifold in the space M(1,4)×R(u).

1. $\langle P_1, P_2\rangle$:

$$F\left(x_0 + x_4, x_3, \left(x_0^2 - x_1^2 - x_2^2 - x_4^2\right)^{1/2}, u\right) = 0;$$

2. $\langle P_3, L_3\rangle$:

$$F\left(x_0 + x_4, \left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0^2 - x_3^2 - x_4^2\right)^{1/2}, u\right) = 0;$$

3. $\langle G, L_3\rangle$:

$$F\left(x_3, \left(x_0^2 - x_4^2\right)^{1/2}, \left(x_1^2 + x_2^2\right)^{1/2}, u\right) = 0;$$

4. $\langle P_3, X_1 \rangle$:

$$F\left(x_2, x_0+x_4, \left(x_0^2-x_3^2-x_4^2\right)^{1/2}, u\right)=0;$$

5. $\langle P_3, X_4 \rangle$:

$$F\left(x_0+x_4, x_1, x_2, u\right)=0;$$

6. $\langle G, X_1 \rangle$:

$$F\left(x_2, x_3, \left(x_0^2-x_4^2\right)^{1/2}, u\right)=0;$$

7. $\langle L_3-P_3, X_4 \rangle$:

$$F\left(x_0+x_4, \left(x_1^2+x_2^2\right)^{1/2}, \arctan\frac{x_1}{x_2}+\frac{x_3}{x_0+x_4}, u\right)=0;$$

8. $\langle L_3+\lambda G, X_3, \lambda>0 \rangle$:

$$F\left(\left(x_0^2-x_4^2\right)^{1/2}, \left(x_1^2+x_2^2\right)^{1/2}, \ln\left(x_0+x_4\right)+\lambda\arctan\frac{x_1}{x_2}, u\right)=0;$$

9. $\langle L_3, X_4 \rangle$:

$$F\left(x_3, x_0+x_4, \left(x_1^2+x_2^2\right)^{1/2}, u\right)=0;$$

10. $\langle L_3, X_0+X_4 \rangle$:

$$F\left(x_3, x_4, \left(x_1^2+x_2^2\right)^{1/2}, u\right)=0;$$

11. $\langle L_3, X_4-X_0 \rangle$:

$$F\left(x_0, x_3, \left(x_1^2+x_2^2\right)^{1/2}, u\right)=0;$$

12. $\left\langle L_3+\frac{1}{2}(P_3+C_3), X_0+X_4 \right\rangle$:

$$F\left(\left(x_1^2+x_2^2\right)^{1/2}, \left(x_3^2+x_4^2\right)^{1/2}, x_1x_4-x_2x_3, u\right)=0;$$

13. $\left\langle L_3+\frac{\lambda}{2}(P_3+C_3), X_0+X_4, 0<\lambda<1 \right\rangle$:

$$F\left(\left(x_1^2+x_2^2\right)^{1/2}, \left(x_3^2+x_4^2\right)^{1/2}, \lambda\arctan\frac{x_1}{x_2}-\arctan\frac{x_3}{x_4}, u\right)=0;$$

14. $\langle L_3, P_3+C_3 \rangle$:

$$F\left(x_0, \left(x_1^2+x_2^2\right)^{1/2}, \left(x_3^2+x_4^2\right)^{1/2}, u\right)=0;$$

15. $\langle X_0+X_4, X_4-X_0 \rangle$:

$$F\left(x_1, x_2, x_3, u\right)=0;$$

16. $\langle X_4, X_1 \rangle$:

$$F\left(x_0+x_4, x_2, x_3, u\right)=0;$$

17. $\langle X_1, X_4-X_0 \rangle$:

$$F\left(x_0, x_2, x_3, u\right)=0;$$

18. $\langle L_3+\alpha(X_0+X_4), X_4, \alpha>0 \rangle$:

$$F\left(x_3, \left(x_1^2+x_2^2\right)^{1/2}, \alpha\arctan\frac{x_1}{x_2}-x_0-x_4, u\right)=0;$$

19. $\langle L_3+\alpha(X_0+X_4), X_4-X_0, \alpha>0 \rangle$:

$$F\left(x_3, \left(x_1^2+x_2^2\right)^{1/2}, x_0-\alpha\arctan\frac{x_1}{x_2}, u\right)=0;$$

20. $\langle L_3+\alpha X_3, X_4, \alpha>0 \rangle$:

$$F\left(x_0+x_4, \left(x_1^2+x_2^2\right)^{1/2}, \alpha\arctan\frac{x_1}{x_2}+x_3, u\right)=0;$$

21. $\langle L_3+\alpha X_3, X_0+X_4, \alpha>0 \rangle$:

$$F\left(x_4, \left(x_1^2+x_2^2\right)^{1/2}, \alpha\arctan\frac{x_1}{x_2}+x_3, u\right)=0;$$

22. $\langle L_3+\alpha X_3, X_4-X_0, \alpha>0 \rangle$:

$$F\left(x_0, \left(x_1^2+x_2^2\right)^{1/2}, \alpha\arctan\frac{x_1}{x_2}+x_3, u\right)=0;$$

23. $\langle L_3+2X_4, X_3 \rangle$:

$$F\left(x_0+x_4, \left(x_1^2+x_2^2\right)^{1/2}, 2\arctan\frac{x_1}{x_2}-x_0+x_4, u\right)=0;$$

24. $\langle G+\alpha X_2, X_1, \alpha>0 \rangle$:

$$F\left(x_3, \left(x_0^2-x_4^2\right)^{1/2}, x_2-\alpha\ln\left(x_0+x_4\right), u\right)=0;$$

25. $\langle P_3 - X_1, X_4 \rangle$:

$$F\left(x_0 + x_4, x_2, x_3 - x_1\left(x_0 + x_4\right), u\right) = 0;$$

26. $\langle P_3 - X_2, X_1 \rangle$:

$$F\left(x_0 + x_4, \frac{x_3}{x_0 + x_4} - x_2, \left(x_0^2 - x_3^2 - x_4^2\right)^{1/2}, u\right) = 0;$$

27. $\langle P_3 - 2X_0, X_4 \rangle$:

$$F\left(x_1, x_2, \left(x_0 + x_4\right)^2 + 4x_3, u\right) = 0;$$

28. $\langle P_3 - 2X_0, X_1 \rangle$:

$$F\left(x_2, \left(x_0 + x_4\right)^2 + 4x_3, x_0 - x_4 + \frac{1}{6}\left(x_0 + x_4\right)^3 + x_3\left(x_0 + x_4\right), u\right) = 0;$$

29. $\langle L_3 - P_3 + 2\alpha X_0, X_4, \alpha > 0 \rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0 + x_4\right)^2 + 4\alpha x_3, 2\alpha \arctan\frac{x_1}{x_2} - x_0 - x_4, u\right) = 0;$$

30. $\langle L_3 + \alpha\left(X_0 + X_4\right), P_3 + C_3 + 2\beta\left(X_0 + X_4\right), \alpha > 0, \beta \geq 0 \rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_3^2 + x_4^2\right)^{1/2}, x_0 - \alpha \arctan\frac{x_1}{x_2} + \beta \arctan\frac{x_4}{x_3}, u\right) = 0;$$

31. $\langle G + \alpha X_3, L_3 + \beta X_3, \alpha > 0, \beta \geq 0 \rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0^2 - x_4^2\right)^{1/2}, \beta \arctan\frac{x_1}{x_2} + x_3 - \alpha \ln\left(x_0 + x_4\right), u\right) = 0;$$

32. $\langle G, L_3 + \alpha X_3, \alpha > 0 \rangle$:

$$F\left(\left(x_0^2 - x_4^2\right)^{1/2}, \left(x_1^2 + x_2^2\right)^{1/2}, \alpha \arctan\frac{x_1}{x_2} + x_3, u\right) = 0;$$

33. $\langle L_3 + 2X_4, P_3 - 2\beta X_0, \beta > 0 \rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0 + x_4\right)^2 + 4\beta x_3, 4\beta \arctan\frac{x_1}{x_2} - 2\beta\left(x_0 - x_4\right) - \frac{1}{3\beta}\left(x_0 + x_4\right)^3 - 2x_3\left(x_0 + x_4\right), u\right) = 0;$$

34. $\langle L_3 + 2X_4, P_3 \rangle$:

$$F\left(x_0 + x_4, \left(x_1^2 + x_2^2\right)^{1/2}, \frac{x_3^2}{x_0 + x_4} + \arctan\frac{x_1}{x_2} + 2x_4, u\right) = 0;$$

35. $\langle L_3, P_3 - 2X_0 \rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0 + x_4\right)^2 + 4x_3, x_0 - x_4 + \frac{1}{6}\left(x_0 + x_4\right)^3 + x_3\left(x_0 + x_4\right), u\right) = 0;$$

36. $\langle P_2, P_1 - X_3 \rangle$:

$$F\left(x_0 + x_4, \left(x_0^2 - x_1^2 - x_2^2 - x_4^2\right)^{1/2}, x_3 - \frac{x_1}{x_0 + x_4}, u\right) = 0;$$

37. $\langle P_1 - X_3, P_2 - \gamma X_2 - \beta X_3, \beta \geq 0, \gamma > 0 \rangle$:

$$F\left(x_0 + x_4, \frac{x_1}{x_0 + x_4} + \frac{\beta x_2}{x_0 + x_4 + \gamma} - x_3, \frac{x_1^2}{x_0 + x_4} + \frac{x_2^2}{x_0 + x_4 + \gamma} + 2x_4, u\right) = 0;$$

38. $\langle P_1, P_2 - X_2 - \beta X_3, \beta \geq 0 \rangle$:

$$F\left(x_0 + x_4, x_3 - \frac{\beta x_2}{x_0 + x_4 + 1}, \frac{x_1^2}{x_0 + x_4} + \frac{x_2^2}{x_0 + x_4 + 1} + 2x_4, u\right) = 0.$$

Now, we present the non-singular manifolds in the space M(1,4)×R(u) invariant under the two-dimensional non-conjugate subalgebras of type A_2 (non-Abelian subalgebras) of the Lie algebra of the group P(1,4).

Proposition 3 [22].

There exist 7 non-conjugate subalgebras of the type A_2 (non-Abelian subalgebras) of the Lie algebra of the group P(1,4).

Consequently, there exist 7 non-singular manifolds in the space M(1,4)×R(u) invariant under the two-dimensional non-conjugate subalgebras of type A_2 of the Lie algebra of the group P(1,4).

Below, for each two-dimensional non-conjugate subalgebra of the type A_2 of the Lie algebra of the group P(1,4), we write its bases elements and corresponding non-singular invariant manifold in the space M(1,4)×R(u).

1. $\langle P_3, -G \rangle$:

$$F\left(x_1, x_2, \left(x_0^2 - x_3^2 - x_4^2\right)^{1/2}, u\right) = 0;$$

2. $\left\langle P_3, -\frac{1}{\lambda} L_3 - G, \lambda > 0 \right\rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \left(x_0^2 - x_3^2 - x_4^2\right)^{1/2}, \ln\left(x_0 + x_4\right) + \lambda \arctan\frac{x_1}{x_2}, u\right) = 0;$$

3. $\langle X_4, -G \rangle$:

$$F\left(x_1, x_2, x_3, u\right) = 0;$$

4. $\left\langle X_4, -\frac{1}{\lambda} L_3 - G, \lambda > 0 \right\rangle$:

$$F\left(x_3, \left(x_1^2 + x_2^2\right)^{1/2}, \ln\left(x_0 + x_4\right) + \lambda \arctan\frac{x_1}{x_2}, u\right) = 0;$$

5. $\langle X_4, -G - \alpha X_1, \alpha > 0 \rangle$:

$$F\left(x_1 - \alpha \ln\left(x_0 + x_4\right), x_2, x_3, u\right) = 0;$$

6. $\left\langle X_4, -\frac{1}{\lambda}\left(L_3 + \lambda G + \alpha X_3\right), \lambda > 0, \alpha > 0 \right\rangle$:

$$F\left(\left(x_1^2 + x_2^2\right)^{1/2}, \ln\left(x_0 + x_4\right) + \lambda \arctan\frac{x_1}{x_2}, \alpha \ln\left(x_0 + x_4\right) - \lambda x_3, u\right) = 0;$$

7. $\langle P_3, -G - \alpha X_1, \alpha > 0 \rangle$:

$$F\left(x_1 - \alpha \ln\left(x_0 + x_4\right), x_2, \left(x_0^2 - x_3^2 - x_4^2\right)^{1/2}, u\right) = 0.$$

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КЛАСИФІКАЦІЯ НЕОСОБЛИВИХ МНОГОВИДІВ У ПРОСТОРІ M(1,4)×R(u), ІНВАРІАНТНИХ ВІДНОСНО ОДНО- І ДВОВИМІРНИХ НЕСПРЯЖЕНИХ ПІДАЛГЕБР АЛГЕБРИ ЛІ ГРУПИ ПУАНКАРЕ P(1,4)

Використовуючи класифікацію одно- і двовимірних неспряжених підалгебр алгебри Лі групи Пуанкаре P(1,4) в класи ізоморфних підалгебр, проведено класифікацію неособливих многовидів у просторі M(1,4)×R(u), які інваріантні відносно цих підалгебр.

Ключові слова: неособливі многовиди, простір Мінковського, група Пуанкаре P(1,4), неспряжені підалгебри алгебри Лі, класифікація підалгебр.

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КЛАССИФИКАЦИЯ НЕОСОБЫХ МНОГООБРАЗИЙ В ПРОСТРАНСТВЕ M(1,4)×R(u), ИНВАРИАНТНЫХ ОТНОСИТЕЛЬНО ОДНО- И ДВУХМЕРНЫХ НЕСОПРЯЖЕННЫХ ПОДАЛГЕБР АЛГЕБРЫ ЛИ ГРУППЫ ПУАНКАРЕ P(1,4)

Используя классификацию одно- и двухмерных несопряженных подалгебр алгебры Ли группы Пуанкаре P(1,4) в классы изоморфных подалгебр, проведена классификация неособых многообразий в пространстве M(1,4)×R(u), которые инвариантны относительно этих подалгебр.

Ключевые слова: неособые многообразия, пространство Минковского, группа Пуанкаре P(1,4), несопряженные подалгебры алгебры Ли, классификация подалгебр.