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Diána Himics[1,a], A.I. Lengyel[2,b], Z.Tarics[2,c], Jolán Turóci[1,d]
[1] Uzhhorod State University, Ukraine
[2] Institute of Electron Physics, Nat. Ac. Sc. of Ukraine, Uzhhorod
[a] e-mail: *himicsdia@hotmail.com*
[b] e-mail: *alexander-lengyel@rambler.ru*
[c] e-mail: *tarics1@rambler.ru*
[d] e-mail: *turoczi.jolika@citromail.hu*

PROTON ELASTIC SCATTERING AT THE LARGE HADRON COLLIDER

At the world biggest accelerator, the Large Hadron Collider (LHC), now operating at CERN (Geneva), protons are boosted to a velocity close to that of light, which means that processes there are absolutely relativistic, and thus all the relevant formalism should be based on non-euclidean geometry, developed almost two centuries ago by Bolyai, Gauss and Lobachevsky. A detailed analysis of proton elastic scattering in the kinematical region of the Large Hadron Collider (LHC) is presented.

Key words: Large Hadron Collider, proton, non-euclidean geometry.

Introduction

At the world biggest accelerator, the Large Hadron Collider (LHC), now operating at CERN (Geneva), protons are boosted to a velocity close to that of light, which means that processes there are absolutely relativistic, and thus all the relevant formalism should be based on non-Euclidean geometry, developed almost two centuries ago by Bolyai, Gauss and Lobachevsky.

The present contribution is an update of recent papers [1], in which elastic proton scattering was studied. In those papers a simple dipole Pomeron (DP) model reproducing the structure of the first and second diffraction cones in pp- and $p\bar{p}$ scattering was developed. The simplicity and transparency of the model enables one to control various contributions to the scattering amplitude, in particular the interplay between the C-even and C-odd components of the amplitude, as well as their relative contribution, changing with *s* and *t*.

The possible extensions of DP model include:

- The dip-bump structure typical to high-energy diffractive processes;
- Non-linear Regge trajectories;
- Possible Odderon (C-odd asymptotic Regge exchange);
- Compatible with s- and t-channel unitarity.

Here we consider the spin less case of the invariant high-energy scattering amplitude, $A(s,t)$, where *s* and *t* are the usual Mandelstam variables. The basic assumptions of the model are: 1. The scattering amplitude is a sum of four terms, two asymptotic (Pomeron (P) and Odderon (O)) and two non-asymptotic ones or secondary Regge pole contributions, where *P* and *f* have positive *C*-parity, thus entering in the scattering amplitude with the same sign in *pp*- and $p\bar{p}$-scattering, while the Odderon and ω have negative C-parity, thus entering pp- and $p\bar{p}$-scattering with opposite signs, as shown below:

$$A(s,t)_{pp}^{\bar{p}p} = A_P(s,t) + A_f(s,t) \pm \\ \pm\left[A_\omega(s,t) + A_O(s,t)\right] \tag{1}$$

where the symbols *P, f, O, ω* stand for the relevant Regge-pole amplitudes and the super(sub)script, evidently, indicate $\bar{p}p(pp)$ scattering with the relevant choice of the signs in the sum (1). 2. We treat the Odderon, the *C*-odd counterpart of the Pomeron on equal footing, differing by its *C*-parity and the values of its parameters (to be fitted to the data). We examined also a fit to *pp* scattering alone, without any Odderon contribution. The (negative) result is presented in Sec. 3. 3. The main subject of our study is the Pomeron and the Odderon, as a double poles, or DP, see [1], lying on a nonlinear trajectory, whose intercept is not equal to one. This choice is motivated by the unique properties of the DP: it produces logarithmically rising total cross sections at unit Pomeron intercept. By letting $\alpha_p(0)>1$, we allow for a faster rise of the total cross section, although the intercept is about half that in the DL model since the double pole (or dipole) itself drives the rise in energy. A supercritical Pomeron trajectory, $\alpha_p(0)>1$ in the DP is required by the observed rise of the ratio σ_{el}/σ_{tot} , or, equivalently, departure form geometrical scaling. The dipole Pomeron produces logarithmically rising total cross sections and nearly constant ratio of σ_{el}/σ_{tot} at unit Pomeron intercept, $\alpha_p(0)=1$. In addition this mild logarithmic increase of σ_{tot} does not supported by the result of the last experiment at LHC for energy 7 TeV $\sigma_{tot}=(98.3\pm 2.8\pm 0.02)\,mb$ [2]. Along with the rise of the ratio σ_{el}/σ_{tot} beyond the SPS energies requires a supercritical DP intercept, $\alpha_p(0)=1+\delta$, where δ is a small parameter $\alpha_P(0)\sim 0.05$.

2. The model

We use the normalization:

$$\frac{d\sigma}{dt}=\frac{\pi}{s^2}\left|A(s,t)\right|^2 \text{ and } \quad \sigma_{tot}=\frac{4\pi}{s}\operatorname{Im}A(s,t)\Big|_{t=0} \quad (2)$$

Neglecting spin dependence, the invariant proton(antiproton)-proton elastic scattering amplitude is that of Eq. (1). The secondary Reggeons are parametrized in a standard way with linear Regge trajectories and exponential residua, where *R* denotes *f* or ω – the principal non-leading contributions to pp or $\bar{p}p$ scattering:

$$A_R(s,t)=a_R e^{-i\pi\alpha_R(t)/2}e^{b_R t}(s,s_0)^{\alpha_R(t)} \quad (3)$$

with handbook slopes $\alpha'_f(t)=0.84$ and $\alpha'_\omega(t)=0.93$. The values of other parameters of the Reggeons are quoted in Table 1. As argued in the Introduction, the Pomeron is a dipole in the j-plane

$$A_P(s,t)=\frac{d}{d\alpha_P}\left[e^{-i\pi\alpha_P/2}G(\alpha_P)(s,s_0)^{\alpha_P}\right]= \\ e^{i\pi\alpha_P(t)/2}(s,s_0)^{\alpha_P(t)}\left[G'(\alpha_P)+(L-i\pi/2)G(\alpha_P)\right] \quad (4)$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P)=-a_P e^{b_P[\alpha_P-1]}, \quad (5)$$

where $G(\alpha_P)$ is recovered by integration, and, as a consequence, the Pomeron amplitude Eq. (4) can be rewritten in the following "geometrical" form (for the details of the calculations see [1] and references therein)

$$A_P(s,t)=i\frac{a_P s}{b_P s_o}\left[r_1^2(s)\,e^{r_1^2(s)[\alpha_P-1]}-\varepsilon_P r_2^2(s)\,e^{r_2^2(s)[\alpha_P-1]}\right], \quad (6)$$

where

$$r_1^2(s)=b_P+L-i\pi/2, \\ r_2^2=L-i\pi/2,\ L\equiv\ln(s/s_0) \quad (7)$$

We use a representative example of the Pomeron trajectory, namely that with a two-pion square-root threshold, Eq. (8), required by t-channel unitarity and accounting for the small-t "break" [16],

$$\alpha_P\equiv\alpha_p(t)=1+\delta_p+\alpha_{1P}t-\alpha_{2P}\left(\sqrt{4m_\pi^2-t}-2m_\pi\right), \quad (8)$$

where m_π – pion mass.

$$A_O(s,t)=\frac{a_O s}{b_O s_0}\left[r_{1O}^2(s)e^{r_{1O}^2(s)[\alpha_O-1]}-\varepsilon_O r_2^2(s)\,e^{r_{2O}^2(s)[\alpha_O-1]}\right], \quad (9)$$

where

$$r_{1O}^2(s)=b_O+L-i\pi/2, \\ r_{2O}^2=L-i\pi/2,\ L\equiv\ln(s/s_0) \quad (10)$$

and

$$\alpha_O\equiv\alpha_O(t)=1+\delta_o+\alpha_{1O}t-\alpha_{2O}\left(\sqrt{4m_\pi^2-t}-2m_\pi\right), \quad (11)$$

The form and properties of Odderon trajectory is the same along with the scale value s_0=100 GeV^2. The adjustable parameters are: $\delta_P, \alpha_{iP}, a_P, b_P, \varepsilon_P$ for the Pomeron and $\alpha_{iO}, a_O, \delta_O, b_O, \varepsilon_O$ for the Odderon. The results of the fitting procedure is presented below.

3. Fitting procedure

The model contains (at most) 16 parameters (depending on the choice of the trajectories) to be fitted to about 1200 data points simultaneously in *s* and *t*. By a straightforward minimization one has little chances to find the solution, because of possible correlations between different contribution and the parameters, including the *P–f* and $O-\omega$ mixing and the unbalanced role of different contributions/data points. To avoid false χ^2 minima, we proceed step-by-step: we first fit the model to the forward data: the total cross section and the ratio $\rho = \mathrm{Re}\,A(s,t=0)/\mathrm{Im}\,A(s,t=0)$, starting with the dominant Pomeron contribution with the sub-leading Reggeons, then we perform the fit for first cone and finally adding the Odderon to the whole region of momentum transfer. The compiled data (see [1]) were used in our fitting procedure. The data are: total *pp* and $\bar{p}p$ cross section measurements spanning energy range from 5 to 7 TeV and to 2.0 TeV, respectively. First of all we check the possible best fit for forward scattering, i.e. fitting the total cross section $\sigma(s)$ and $\rho(s)$ for a well established set of this type of data plus the new measurement at 7 TeV [2]. We perform also a fit to *pp* data alone, see the previous Section, to see whether the observed dynamics of dip can be reproduced by the Pomeron alone. The contribution to the global χ^2 from tiny effects, such as the small $-|t|$ "break" in the first (and second) cone, possible oscillations in the slope of the cone(s) etc. should not corrupt the study of the dynamics in the dip-bump region. The differential elastic scattering cross section cover the momentum transfer range $|t|$=0.05–15 GeV^2. Next, we included in the fit the differential cross sections in first cone chosen, somewhat subjectively for $|t| \le 0.5$ GeV^2 along with forward data, to determine the remaining parameters of the Reggeons and the Pomeron, b_f, b_ω for Reggeons, a_P, b_P, α_{1P} and α_{2P} for the Pomeron, important in first cone. Among the parameters of the previous fit we fixed the parameters responsible for rise of the total cross section.

Table 1

Parameters, quality of the fit and predictions of σ_{tot} obtained in the whole interval in *s* and *t*

Parameter	Value	Error
a_P	269	5
b_P, GeV^{-2}	6.93	0.06
α_{1P}, GeV^{-2}	0.474	0.006
α_{2P}	0.0060	0.0010
δ_P	0.0504	0.0031
ε_P	0.0167	0.0005
a_O	0.160	0.006
b_O, GeV^{-2}	1.79	0.09
α_{1O}, GeV^{-2}	0.276	0.008
α_{2O}	0.339	0.017
δ_O	0.106	0.008
ε_O	-0.223	0.032
a_f	-13.2	0.1
α_f	0.790	0.004
b_f, GeV^{-2}	4.24	0.15
a_ω	8.51	0.32
α_ω	0.473	0.012
b_ω, GeV^{-2}	15	fixed
χ^2/dof	3.55	
$\sigma_{tot}(7TeV)$	98.1 ± 0.1	
σ_{tot} (14TeV)	111.4 ± 0.1	

5. Conclusions

A basic problem in studying the Pomeron and Odderon is their identification i.e. their discrimination from other contributions. Although this procedure is model-dependent, we try to do this possibly in a general way. The aim of the present paper was to trace the Pomeron and Odderon contribution under conditions accessible within LHC kinematics. This was feasible due to the simplicity of the model, which has the important property of reproducing itself (approximately) against unitarity (absorption) corrections, that are small anyway.

We have presented the "minimal version" of the DP model. It can be further extended, refined and improved, while its basic features

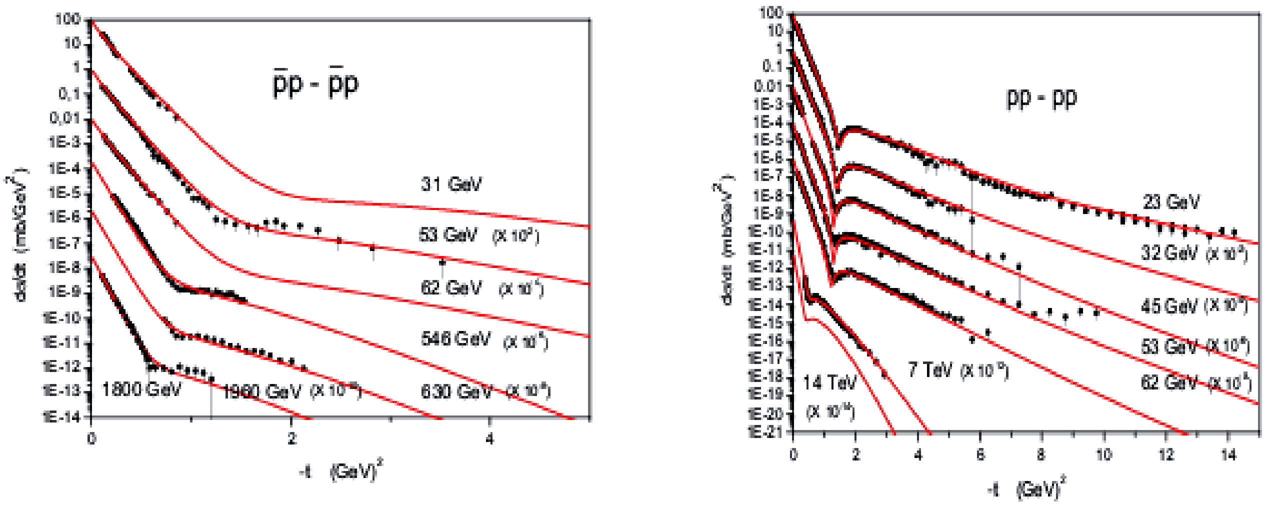

Figure 1. Differential $\bar{p}p$ (a) and pp (b) cross sections calculated from the model, Eqs. (2)-(9) with the Odderon term (10)-(12) and fitted to the data in the range $-t = 0.1-15\,GeV^2$.

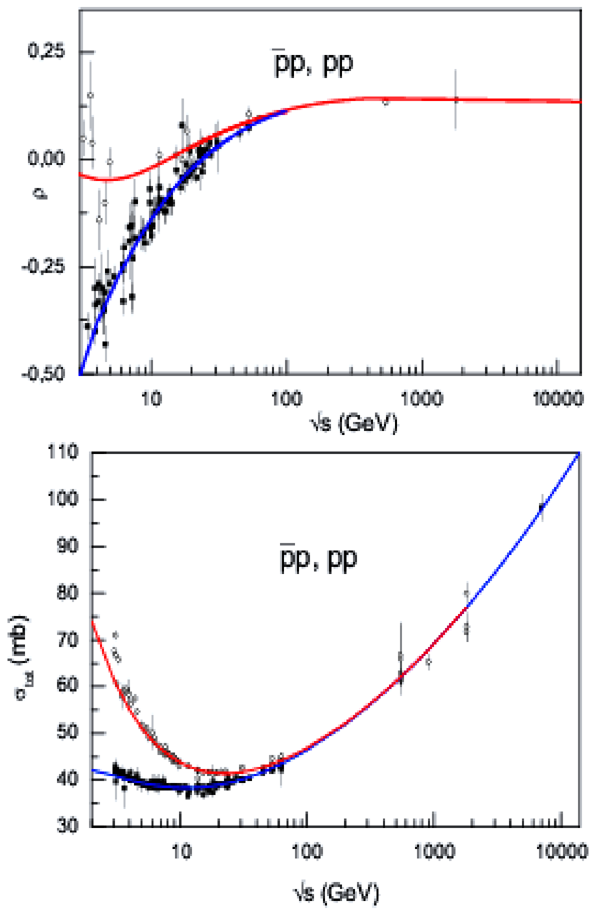

Figure 2. (a) Ratio of the real to imaginary part for pp and $p\overline{p}$ scattering amplitude calculated from the model. (b) pp and $p\overline{p}$ total cross sections calculated from the model, Eq. (2)- (9), and fitted to the data in the range $\sqrt{s} = 5\,GeV - 7\,TeV$ model. The red curve presents the $p\overline{p}$ calculation, the blue one corresponds to pp.

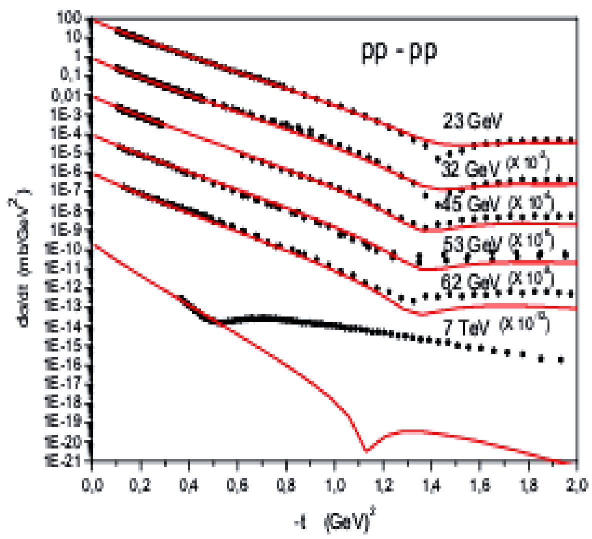

Figure 3. Differential pp cross sections fitted without the Odderon term to the ISR data, calculated from the model Section 2.

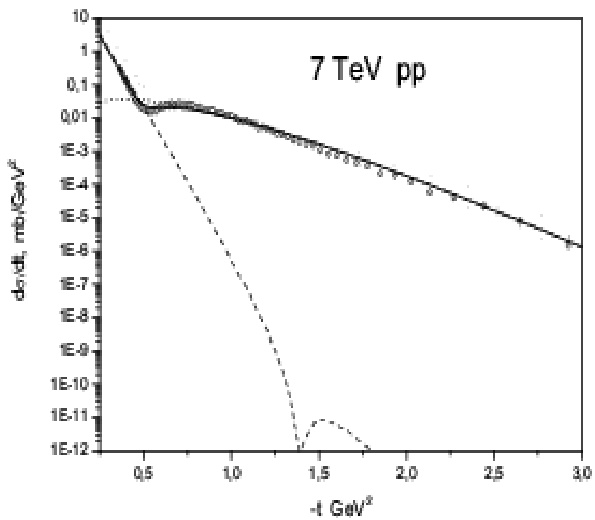

Figure 4. pp differential cross section at 7 TeV calculated from the model. The dashed line corresponds to the Pomeron contribution, the dotted line to that of the Odderon, and solid line to the differential cross section.

remain intact. The anticipated rescaling of the LHC energy down to that of the highest Teavatron energy may provide a definite answer to the questions concerning the Odderon in pp vs. $p\bar{p}$ scattering, raised in the present paper.

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Д. Хімич[1], О.І. Лендел[2], З.Торич[2], Й. Туроці[1]
[1] Ужгородський національний університет, Україна
[2] Інститут електронної фізики, НАН України, Ужгород

ПРУЖНЕ РОЗСІЯННЯ ПРОТОНІВ У ВЕЛИКОМУ АДРОННОМУ КОЛАЙДЕРІ

У найбільшому в світі прискорювачі Large Hadron Collider (LHC), ЦЕРН, Женева, протони прискорюються до швидкості, близької до швидкості світла. Таким чином, процеси в ньому абсолютно релятивістські, тому всі відповідні формалізми повинні бути ґрунтовані на неевклідовій геометрії, розробленій майже два століття тому Бояі, Гауссом і Лобачевським. Представлено детальний аналіз пружного розсіяння протонів в кінематичній області Великого адронного колайдера.

Ключові слова: великий адронний колайдер, протони, неевклідова геометрія.

Д. Химич[1], А.И. Лендел[2], З.Торич[2], И. Туроци[1]
[1] Ужгородский национальный университет, Украина
[2] Институт электронной физики, НАН Украины, Ужгород

УПРУГОЕ РАССЕЯНИЕ ПРОТОНОВ В БОЛЬШОМ АДРОННОМ КОЛЛАЙДЕРЕ

В крупнейшем в мире ускорителе Large Hadron Collider (LHC), ЦЕРН, Женева, протоны ускоряются до скорости, близкой к скорости света. Таким образом, процессы в нем абсолютно релятивистские, поэтому все соответствующие формализмы должны быть основаны на неевклидовой геометрии, разработанной почти два века назад Бояйи, Гауссом и Лобачевским. Представлен подробный анализ упругого рассеяния протонов в кинематической области Большого адронного коллайдера.

Ключевые слова: большой адронный коллайдер, протоны, неевклидовая геометрия.